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Free Vibration Analysis of Single-Layered Graphene Sheets Based on a Continuum Model

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Abstract

Recent experiments have shown the applicability of single-layered graphene sheets as electromechanical resonators. Here we introduce a nonlinear continuum plate model for the study of linear and nonlinear free vibration properties of single-layered graphene sheets. By this analysis, natural frequencies and the associated mode shapes pertaining to different values of m and n for single-layered graphene sheets are given. Also, the effects of the aspect ratio on the fundamental frequencies and the associated mode shapes are investigated. The nonlinear relationship between vibration amplitude and frequency are analyzed in the neighborhood of a given frequency. Parts of results are verified well with experimental observations.

Keywords: Single-layered Graphene Sheet; Vibration Properties; Continuum Model

1 INTRODUCTION

Since Novoselov et al. [1, 2] successfully separated thin graphene sheet(GS) out of bulk graphite by mechanical exfoliation of highly ordered pyrolytic graphite by Scotch tape in 2004, it have stimulated great interest in studying various fundamental properties and possible applications of grpahene sheets. He and his coauthors [3-5] analyzed vibration properties of multi-layered graphene sheets(MLGSs) with or without an elastic matrix using a continuum model including a formula explicitly derived for van der Waals(vdW) interaction. They found that the effect of vdW interaction had no influence on the lowest natural frequency of multilayered graphene sheets without an elastic matrix but played a significant role in all higher natural frequencies. They also studied the influence of vdW interaction and the moduli of the surrounding matrix on the natural frequencies. Behfar and Naghdabadi [6] investigated vibration of multi-layered graphene sheets embedded in polymer matrices, in which an anisotropic plate model for polymer molecules was used through vdW interactions. They derived the natural frequencies and the associated vibration mode shapes of this composite system. Bunch et al. [7] reported the fundamental frequencies and the quality factors of graphene resonators fabricated from single- and multi-layered graphene sheets by mechanically exfoliating thin sheets from graphite suspended over a SiO_2 trench. Sakhaee-Pour et al. [8] studied mode shapes and natural frequencies of single-layered graphene sheets(SLGSs) using a molecular structural mechanics method developed by Li and Chou [9]. The discerned aspect ratio is important at higher modes, but the modes themselves do not have an impact on the fundamental frequencies. Recently, Shen et al.[10] studied nonlinear vibration behavior of single-layered graphene sheet in thermal environments via nonlocal orthotropic plate model. Wang et al.[11] investigated nonlinear vibration properties of multi-walled graphene sheet by a nonlinear continuum model, in which the influence of van der Waals force on vibration properties of MLGS as well as vibration models were analyzed.

It is shown that the classical continuum models are efficient in vibration analysis of graphene sheets. However, it can be found that most studies have focused on linear vibration properties of single- or multi-layered graphene sheets. Thus, in this paper, nonlinear and linear free vibration properties of single-layered graphene sheet are studied using a nonlinear continuum mechanical model described via the von kármán equation of motion of plates. Based on the present analysis, linear and nonlinear vibration modes and frequencies are given for SLGSs. The influences of aspect rations on the fundamental frequencies as well as the backbone curves are also proposed for SLGS.

2 THEORETICAL MODEL

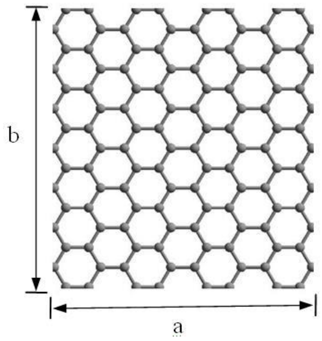

FIG.1 A SINGLE LAYERED GRAPHENE SHEET

Consider a SLGS as a finite, simply supported rectangle plate is shown in Fig.1. The length of this plate is a, the width b, and the thickness is h. Equation governing the free vibration of SLGS can be described via the von kármán equation, i.e.

$$D\nabla^4 w + \rho h \frac{\partial^2 w}{\partial t^2} = \frac{\partial^2 F}{\partial y^2}\frac{\partial^2 w}{\partial x^2} - 2\frac{\partial^2 F}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 F}{\partial x^2}\frac{\partial^2 w}{\partial y^2} \tag{1}$$

where x and y are the coordinates in length and width sides, w is the deflection of the sheet, t is the time, D is the bending stiffness of the individual sheet, and ρ is the mass density of GS. F is an in-plane Airy stress function, given by the following relation:

$$\frac{1}{Eh}\nabla^4 F = \left[(\frac{\partial^2 w}{\partial x \partial y})^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} \right] \tag{2}$$

In Eqs. (1) and (2), the bi-harmonic operator is defined as

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \tag{3}$$

It is noted that the present nonlinear model can be reduced to the linear model presented as all nonlinear terms are ignored in Eq. (1). Thus, the present model is available for nonlinear as well as linear vibration analysis of SLGS.

For the boundary condition of simply supported with immovable edges, the deflection w of SLGS can be approximated by a periodic solution that satisfies all boundary conditions identically, as follows.

$$w(x,y,t) = \sum_{m=1}^{M}\sum_{n=1}^{N} A_{mn}(t) Sin(\frac{m\pi x}{a}) Sin(\frac{n\pi y}{b}) \tag{4}$$

where A_{mn} are the generalized coordinates (unknown functions of t), and a and b are length and width of SLGS, respectively, and m and n are numbers of half-waves in x and y directions, respectively. In the following analysis, $m=1$ and $n=1,3$ are included in the expansion for studying nonlinear vibrations of SLGS in the spectral neighborhood of lower resonances. Substituting the expansion of w into Eqs. (1) and (2) and using the Galerkin method, two second-order, ordinary, coupled nonlinear differential equations will be obtained with variables $A_{11}(t)$ and $A_{13}(t)$ as below.

$$\begin{aligned} &\frac{1}{4}ab\rho h\ddot{A}_{11} + \frac{B_1\pi^4}{2ab}A_{11} + \frac{B_2\pi^4}{2ab}A_{11} + \frac{aD\pi^4}{4b^3}A_{11} + \frac{D\pi^4}{2ab}A_{11} \\ &+\frac{bD\pi^4}{4a^3}A_{11} - \frac{B_2\pi^4}{2ab}A_{13} + \frac{2B_3\pi^4}{ab}A_{13} - \frac{B_5\pi^4}{ab}A_{13} \\ &+\frac{B_6\pi^4}{ab}A_{13} + \frac{B_7\pi^4}{ab}A_{13} - \frac{B_8\pi^4}{4ab}A_{13} = 0 \end{aligned} \tag{5}$$

and

$$\frac{1}{4}ab\rho h\ddot{A}_{13}+\frac{9B_1\pi^4}{2ab}A_{13}+\frac{9B_4\pi^4}{2ab}A_{13}+\frac{81aD\pi^4}{4b^3}A_{13}+\frac{9D\pi^4}{2ab}A_{13}$$
$$+\frac{bD\pi^4}{4a^3}A_{13}-\frac{B_2\pi^4}{2ab}A_{11}+\frac{2B_3\pi^4}{ab}A_{11}-\frac{B_5\pi^4}{4ab}A_{11} \tag{6}$$
$$+\frac{B_6\pi^4}{ab}A_{11}+\frac{B_7\pi^4}{ab}A_{11}-\frac{B_8\pi^4}{4ab}A_{11}=0$$

where the expressions of $B_1, B_2, \cdots, B_8$ are presented in Appendix A

Then, the harmonic balance method is applied to resolve equations (5) and (6), in which the steady state response can be written as

$$A_{mn}(t)=c_{mn}\cos(\omega t) \tag{7}$$

where ω is the vibration frequency, and c_{mn} are the vibration amplitude of SLGS corresponding to mode (m, n). Substituting Eq. (7) into Eqs. (5) and (6), the resulting equations of motion in the frequency domain are obtained as follows:

$$H_1c_{11}+H_2(c_{11})^3+H_3(c_{11})^2c_{13}+H_4c_{11}(c_{13})^2=0 \tag{8}$$

and

$$H_5c_{13}+H_6(c_{11})^3+H_7(c_{11})^2c_{13}+H_8(c_{13})^3=0 \tag{9}$$

where the expressions of $H_1, H_2, \cdots, H_8$ are presented in Appendix A. It can be found that the amplitude-frequency relationship for the nonlinear free vibration of SLGS by solving Eqs. (8) and (9) iteratively.

3 Discussion and Results

In present analysis, Young's modulus of SLGS $E=1.02TPa$, Poisson's ratio $\nu=0.16$ [12], and the mass density $\rho =$ 2250 kg/m^3, thickness of SLGS is assumed to be $h=0.34nm$.

3.1 Linear Free Vibration Analysis for SLGS

For analyzing the linear free vibration properties of SLGS, nonlinear terms in Eq. (1) need be ignored. The deflection of SLGS is approximated by a periodic solution of the form:

$$w(x,y,t)=ASin(\frac{m\pi x}{a})Sin(\frac{n\pi y}{b})e^{i\omega t} \tag{10}$$

Thus, substituting Eq. (10) into Eq. (1), we can obtain the natural frequencies of SLGS with different (n, m) as well as the influence of the aspect ratio of SLGS on its free vibration properties.

Table 1 shows the natural frequencies for a square SLGS with width of 10 nm corresponding to the different combinations of m and n. According to $\omega_1<\omega_2<\cdots<\omega_{10}$, it can found from Table 1 that the first ten natural frequencies of SLGS with a=b=10 nm are $\omega_1=0.0691THz$, $\omega_2=0.1728THz$, $\omega_3=0.2765THz$, $\omega_4=0.3456THz$, $\omega_5=0.4493THz$, $\omega_6=0.5875THz$, $\omega_7=0.6221THz$, $\omega_8=0.6912THz$, $\omega_9=0.8640THz$, $\omega_{10}=0.8986THz$, respectively. It is also found that the fundamental (first) frequency is 69.1 MHz, which agrees well with the corresponding experimental result $f_0=70.5$ MHz [7].

TABLE 1 NATURAL FREQUENCIES (THZ) FOR A SQUARE SINGLE-LAYERED GRAPHENE SHEET WITH WIDTH A=B = 10 NM

n	m=1	m=2	m=3	m=4	m=5	m=6
1	0.0691	0.1728	0.3456	0.5875	0.8986	1.2786
2	0.1728	0.2765	0.4493	0.6912	1.0023	1.3824
3	0.3456	0.4493	0.6221	0.8640	1.1751	1.5552
4	0.5875	0.6912	0.8640	1.1060	1.4170	1.7972
5	0.8986	1.0023	1.1751	1.4170	1.7280	2.1082
6	1.2788	1.3824	1.5552	1.7972	2.1082	2.4884
7	1.7280	1.8317	2.0045	2.2465	2.5575	2.9377
8	2.2465	2.3501	2.5229	2.7649	3.0759	3.4561
9	2.8340	2.9377	3.1105	3.3524	3.6635	4.0436
10	3.4906	3.5943	3.7671	4.0091	4.3201	4.7003

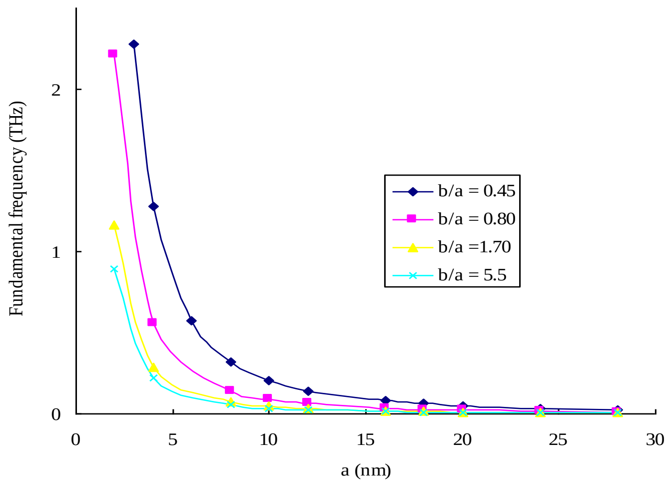

FIG. 2 FUNDAMENTAL FREQUENCIES of SLGSs with DIFFERENT ASPECT RATIOS B/A

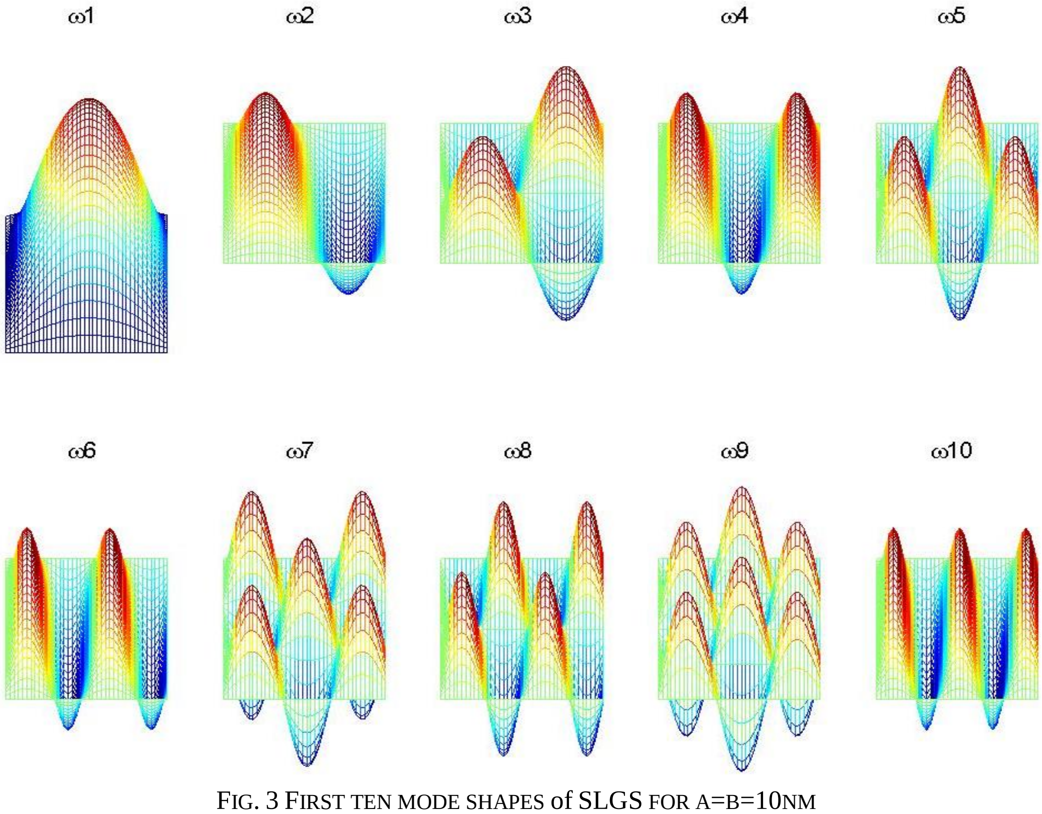

FIG. 3 FIRST TEN MODE SHAPES of SLGS FOR A=B=10NM

In Fig.2, the fundamental frequencies with different aspect ratios are given. Given a special aspect ratio, it is observed that the fundamental frequencies are reduced monotonously with increasing of a. For a fixed a, the fundamental frequencies are increased with reducing of the aspect ratios. Fig. 2 shows that the fundamental frequencies of SLGSs are of the order of 2.2GHz—3.0THz. It will indicate that SLGSs would be employed as resonators with broader and higher ranges of sensitivities. In Fig. 3, the first ten mode shapes for a square SLGS with width a=b=10 nm are presented. It is clear that the bending modes are the dominant ones when SLGSs are vibrated freely.

3.2 Nonlinear Free Vibration Analysis for SLGS

Here, the nonlinear free vibration properties of SLGSs with $a = b = 10$ nm are analyzed in the following. It is very important for investigating the nonlinear free vibration of SLGS system because it determines the dynamic characteristics of the system which can be defined by the amplitude-frequency relations and modes of vibration. When the frequency ω changes respectively in the neighborhood of $\omega_{11} = \omega(n=1, m=1) = 0.0691$ THz and $\omega_{13} = \omega(n=1, m=3) = 0.3456$ THz, the total responses of amplitude of SLGSs are shown in Figs. (4a) and (4b). The horizontal axial represents the dimensionless frequency with ω/ω_{11} and ω/ω_{13}. It can be found from Fig. 4 that the total mode amplitudes of SLGSs are similar and increase monotonously with increasing the frequency in the neighborhood of $\omega_{11} = 0.0691\,THz$ and $\omega_{13} = 0.3459\,THz$. Furthermore, nonlinear modes of SLGS at $\omega/\omega_{11} = 1.18$ with increments of $T/24$ for half-a-cycle are given in Fig. 5, where $T = 2\pi/\omega$. From Figure 5, it can be displayed clearly the vibration process of SLGS at the current frequency.

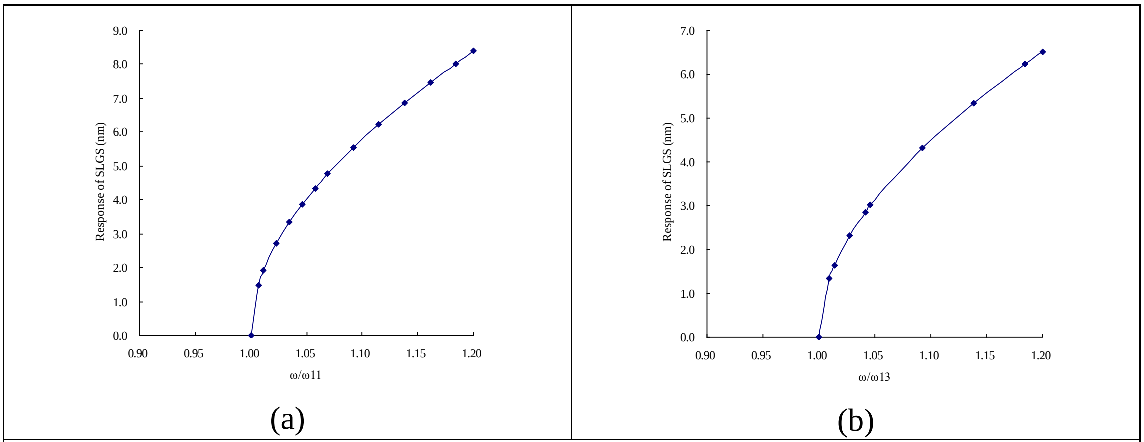

FIG. 4 RESPONSE of SLGS in the NEIGHBORHOOD of $\omega_{11} = 0.0691\,THz$ (A) AND $\omega_{13} = 0.3456\,THz$ (B)

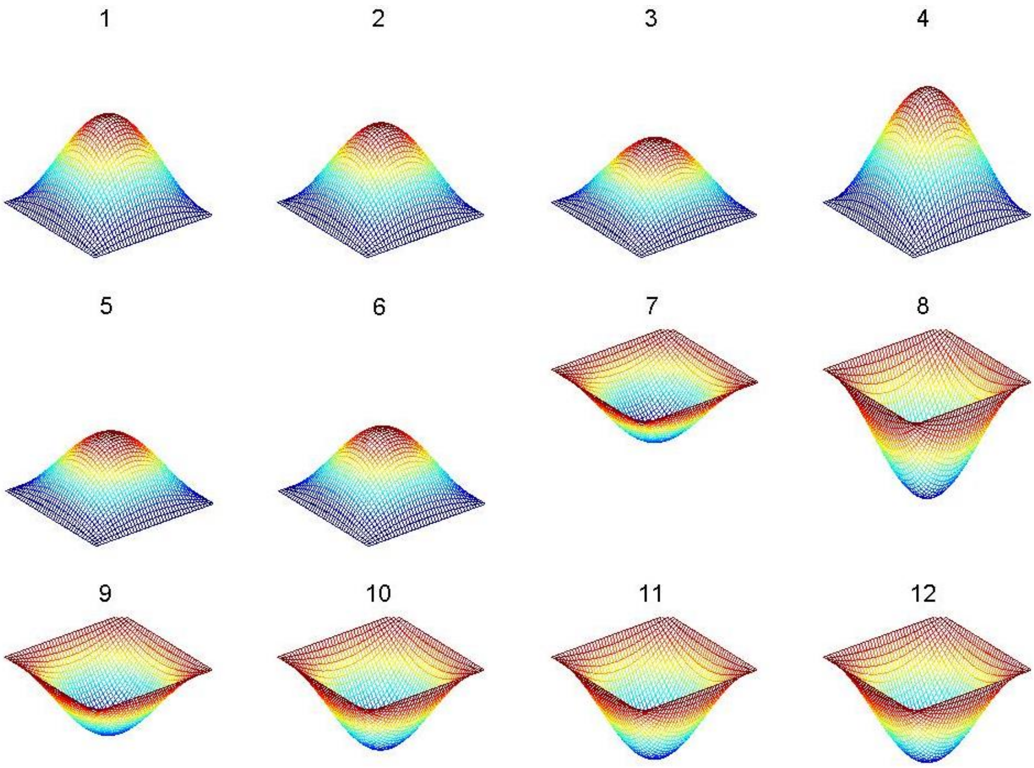

FIG. 5 VIBRATION of SLGS DURING HALF-A-CYCLE at POINT $\omega/\omega_{11} = 1.18$

4 CONCLUSIONS

Based on a nonlinear continuum plate model, the linear and nonlinear free vibration properties of single-layered graphene sheets are investigated. Firstly, the natural frequencies and the associated linear mode shapes of SLGSs are obtained with the combination of m and n . Besides, the influences of the different aspect ratios on the fundamental frequencies of SLGS are explored. Furthermore, the nonlinear vibration properties of SLGS are also analyzed in the neighborhood of a given frequency. Parts of results are verified well with experimental observations.

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REFERENCES

[1] K. S. Novoselov, A. K. Geim, S. V. Morozov, D. Jiang, Y. Zhang, S. V. Dubonos, I. V. Grigorieva and A. A. Firsov, "Electric Field Effect in Atomically Thin Carbon Films." Science 306 (2004), 666-669

[2] K. S. Novoselov, D. Jiang, F. Schedin, T. J. Booth, T. T. Khotkevich, S. V. Morozov and A. K. Geim, "Two-dimensional atomic crystals." Proc. Nat. Acad. Sci. USA 102 (2005), 10451-10453

[3] X. Q. He, S. Kitipornchai and K. M. Liew, "Resonance analysis of multi-layered graphene sheets as nanoscale resonators." Nanotechnology 16 (2005), 2086-2091

[4] S. Kitipornchai, X. Q. He and K. M. Liew, "Continuum model for the vibration of multi layered graphene sheets." Phys. Rev. B 72 (2005), 075443

[5] K. M. Liew, X. Q. He and S. Kitipornchai, "Predicting nanovibration of multi-layered graphene sheets embedded in an elastic matrix." Acta Materialia 54 (2006), 4229-4236

[6] K. Behfar and R. Naghdabadi, "Nanoscale vibrational analysis of a multi-layered graphene sheet embedded in an elastic medium." Compos. Sci. Technol. 65 (2005), 1159-1164

[7] J. S. Bunch, A. M. van der Zande, S. S. Verbridge, I. W. Frank, D. M. Tanenbaum, J. M. Parpia, H. G. Craighead and P. L. McEuen, "Electromechanical Resonators from Graphene Sheets." Science 315 (2007), 490-493

[8] A. Sakhaee-Pour, M. T. Ahmadian and R. Naghdabadi, "Vibrational analysis of single-layered graphene sheets." Nanotechnology 19 (2008), 085702

[9] C. Li and T. W. Chou, "A structural mechanics approach for the analysis of carbon nanotubes." Int. J. Solids Struct. 40 (2003), 2487-2499

[10] L. Shen, H. S. Shen, C. L. Zhang, "Nonlocal plate model for nonlinear vibration of single layer graphene sheets in thermal environments." Comput. Mater. Sci. 48 (2010), 680-685

[11] J. B. Wang, X. Q. He, S. Kitipornchai, H. W. Zhang, "Geometrical nonlinear free vibration of multi-layered graphene sheets." J. Phys. D: Appl. Phys. 44 (2011), 135401

[12] R.Saito, G.Dresselhous, M. S.Dresselhous, eds. Physical properties of carbon nanotubes. London: Imperial College Press, 1998

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Appendix A

Functions of time B_1 , $B_2, \cdots, B_8$ of Eqs (5) and (6) have long expressions, which are given herein below:

$$B_1 = \frac{a^2\left[(A_{11})^2 + 9(A_{13})^2\right]}{32b^2} \tag{A.1}$$

$$B_2 = \frac{b^2\left[(A_{11})^2 - 2A_{11}A_{13}\right]}{32a^2} \tag{A.2}$$

$$B_3 = \frac{b^2 A_{11}A_{13}}{64a^2} \tag{A.3}$$

$$B_4 = \frac{b^2(A_{13})^2}{288a^2} \tag{A.4}$$

$$B_5 = -\frac{a^2b^2A_{11}A_{13}}{32(4a^2+b^2)^2} \tag{A.5}$$

$$B_6 = \frac{a^2b^2A_{11}A_{13}}{8(a^2+b^2)^2} \tag{A.6}$$

$$B_7 = \frac{a^2b^2A_{11}A_{13}}{8(a^2+b^2)^2} \tag{A.7}$$

$$B_8 = -\frac{a^2b^2A_{11}A_{13}}{32(4a^2+b^2)^2} \tag{A.8}$$

In Eqs (8) and (9), $H_1, H_2, \cdots, H_8$ are expressed as

$$H_1 = \frac{aD\pi^4}{4b^3} + \frac{D\pi^4}{2ab} + \frac{bD\pi^4}{4a^3} - \frac{1}{4}abh\rho\omega^2 \tag{A.9}$$

$$H_2 = \frac{3a\pi^4}{256b^3} + \frac{3b\pi^4}{256a^3} \tag{A.10}$$

$$H_3 = -\frac{9b\pi^4}{256a^3} \tag{A.11}$$

$$H_4 = \frac{27a\pi^4}{256b^3} + \frac{3b\pi^4}{64a^3} + \frac{3ab\pi^4}{16(a^2+b^2)^2} + \frac{3ab\pi^4}{256(4a^2+b^2)^2} \tag{A.12}$$

$$H_5 = \frac{81aD\pi^4}{4b^3} + \frac{9D\pi^4}{2ab} + \frac{bD\pi^4}{4a^3} - \frac{1}{4}abh\rho\omega^2 \tag{A.13}$$

$$H_6 = -\frac{3b\pi^4}{256a^3} \tag{A.14}$$

$$H_7 = \frac{3b\pi^4}{64a^3} + \frac{27a\pi^4}{256b^3} + \frac{3ab\pi^4}{16(a^2+b^2)^2} + \frac{3ab\pi^4}{256(4a^2+b^2)^2} \tag{A.15}$$

$$H_8 = \frac{243a\pi^4}{256b^3} + \frac{3b\pi^4}{256a^3} \tag{A.16}$$