Applied Physics Frontier
August 2014, Volume 2, Issue 2, PP.8-11

Dynamics Analysis of Hyperchaotic Circuit

Jiali Leng*[#]*, Yongming Cao, Kefeng Zhao
Institute for Nonlinear Circuits and Systems of Chongqing University of Posts and Telecommunications, Chongqing 400065, China
[#]Email: wlengjl@163.com

Abstract

This paper proposes a new simple 4D memristive circuit. We find that the system exist richly complex dynamics, such as chaos and hyperchaos. Further numerical study has verified the existence of a hyperchaotic attractor in the memristive circuit.

Keywords: Memristor; Bifurcation; Hyperchaos

1 INTRODUCTION

The memristor predicted by Leon O. Chua [1] is recognized as the fourth fundamental electronic element besides the well-known resistor, capacitor and inductor. The memristor is normally nonlinear element [1-4], which often lead to complex phenomena, and is growing interest in studying nonlinear dynamics in memristive circuits.

In order to better understand the nonlinearity of memristor element, the nonlinear behavior of memristive circuit (especially chaotic behaviour) gradually becomes a focus in recent years, [5-7]. By using different way to replace Chua's diodes of Chua's circuit with a memristor, researchers generate different chaotic circuit.Itoh and Chua replace the Chua's diodes of Chua's circuit with a memristor a one-dimensional (1D) nonautonomous circuit proposed by Driscoll et al. [11], a 3D circuit invented by Muthuswamy and Chua [12], a number of 4D circuits studied by Itoh, Chua, et al. [13-17], and higher dimensional systems presented by Chua and Buscarino, et al. [18 19], etc.So, this paper proposes a new simple 4D memristive circuit, by theoretical derivation and numerical analysis ,we confirme that system existe the hyper-chaotic phenomena.

2 HYPERCHAOS BASED ON CANONICAL CHUA'S CHAOTIC CIRCUIT

2.1 Mathematical model

Canonical Chua's chaotic circuit is improved, using memristor of a piecewise linear function instead of chua's chaotic circuit of resistance R, get a new circuit as shown in Fig. 1.

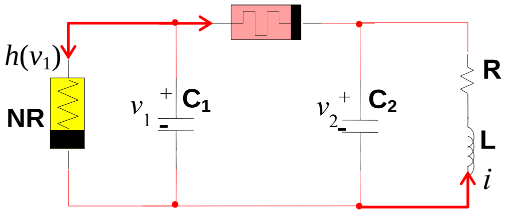

FIG. 1 HYPERCHAOTIC CIRCUIT

We use a piecewise linear function to describe memristor 's equation:

$$W(\phi)=\frac{dq(\phi)}{d(\phi)}=\frac{a}{2b}\left\|\phi+b\right|-\left|\phi-b\right\| \tag{1}$$

a , b is constant.The quations of a new four-dimensional chua's circuit with memristor show below:

$$\begin{cases} C_1\dot{v}_1=W(\phi)(v_2-v_1)-h(v_1) \\ C_2\dot{v}_2=W(\phi)(v_1-v_2)+i \\ L\dot{i}=-v_2-R_0 i \\ \dot{\phi}=v_2-v_1 \end{cases} \tag{2}$$

Set $x=\frac{v_1}{B_p}$, $y=\frac{v_2}{B_p}$, $z=\frac{Ri}{B_p}$, $\tau_0=RC_2$, $w=\frac{\phi}{\tau_0 B_p}$, $\tau=\frac{t}{\tau_0}$, $\alpha=\frac{C_2}{C_1}$, $\beta=\frac{R\tau_0}{L}$, $\gamma=\frac{R_0\tau_0}{L}$, $W(w)=RW(\phi)$, $h(x)=R\frac{h(v_1)}{B_p}$, $h(x)=m_1x+0.5(m_0-m_1)\left(|x+1|-|x-1|\right)$. So, (2) type of state equation can be rewritten:

$$\begin{cases}\dot{x}=\alpha\left[\bar{W}(w)(y-x)-\bar{h}(x)\right]\\ \dot{y}=\bar{W}(w)(x-y)+z\\ \dot{z}=-\beta y-\gamma z\\ \dot{w}=y-x\end{cases} \tag{3}$$

Where $\alpha=15.6$, $\beta=28$, $\gamma=0.315$, $m_0=-1$, $m_1=-0.8$, $a=1$, $b=1$, the hyperchaotic attractor of the system are shown in Fig. 2.

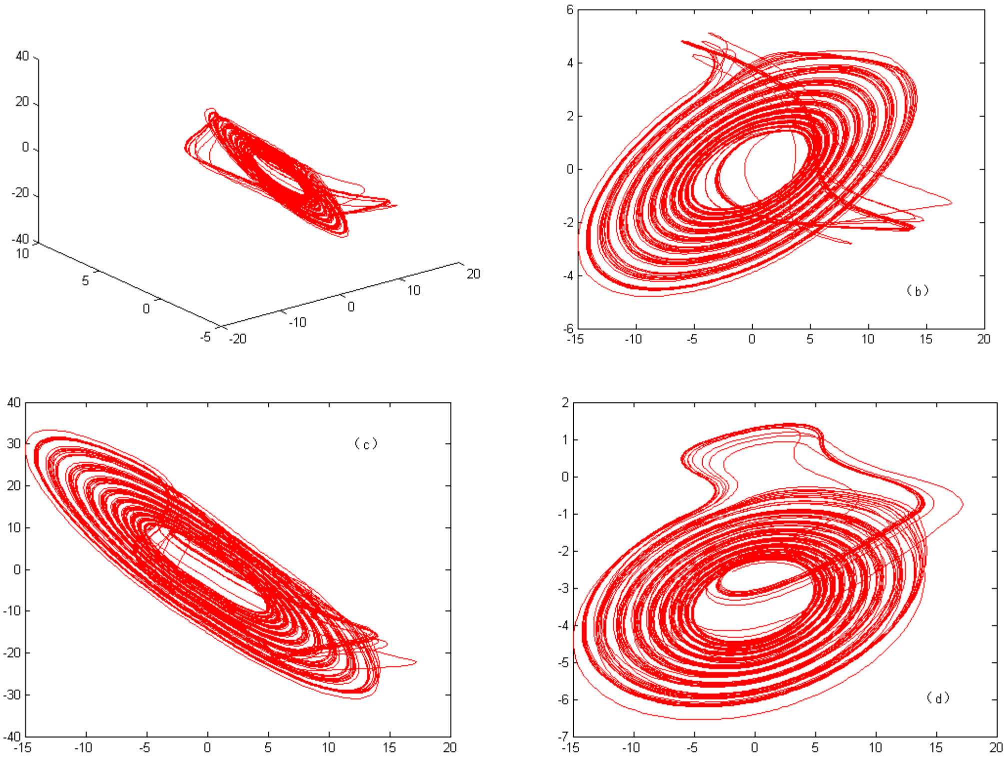

FIG. 2 HYPERCHAOTIC ATTRACTOR (A)X-Y-Z; (B)X-Y; (C)X-Z; (D)X-W

2.2 Basic dynamic behaviour analysis

When $x=y=z=0$, we can solve that all of the system equilibrium is on the Z-axis. Then,

$$O=\{(x,y,z,w)\,|\,x=y=z=0, w=c\in R\}$$

The Jacobian matrix at the equilibrium set O is given by

$$J_O=\begin{bmatrix}\alpha(\bar{W}(c)+\bar{m}_0) & \alpha\bar{W}(c) & 0 & 0\\ \bar{W}(c) & -\bar{W}(c) & 1 & 0\\ 0 & -\beta & -\gamma & 0\\ -1 & 1 & 0 & 0\end{bmatrix} \tag{4}$$

By solving the characteristic eqution $|\lambda \mathrm{E}-J|=0$, we can obtain the Jacobian eigenvalues ,Since $\bar{W}(c)$ is function is a piecewise linear,

- When $|c|\ge b=3.5$, $\bar{W}(c)=a=2.5$ is a constant, we can get: $\lambda_1=0$, $\lambda_2=-60.2153$,

$\lambda_{3,4} = 0.3077 \pm 5.8231i$

- When $-3.5 < c < 3.5$, we find the Jacobian eigenvalues is very complex, When $c \in (0.45 - 0.55)$, We can determine the stability. And when $c \in (-3.5 - 0.45) \cup (0.55 - 3.5)$, this interval is Unstability.

2.3 Lyapunov exponent and Bifurcations.

To verify the hyperchaoticity of system, its Lyapunov exponents are denoted by $LE1 > 0$, $LE2 > 0$, $LE3 = 0$, $LE4 < 0$.

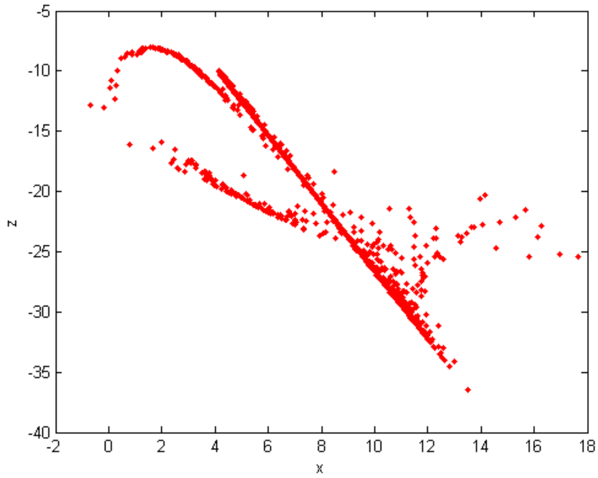

FIG. 3 POINCAR`E MAP OF $x - z$ PLANE

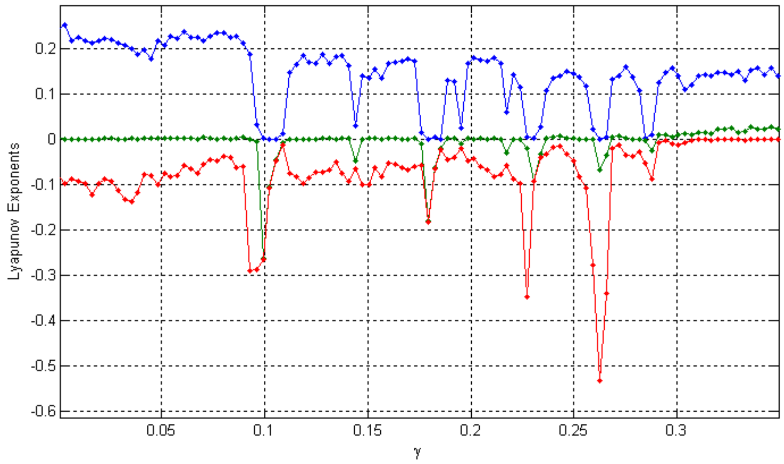

FIG.4 LYAPUNOV EXPONENTS VERSUS THE PARAMETER γ

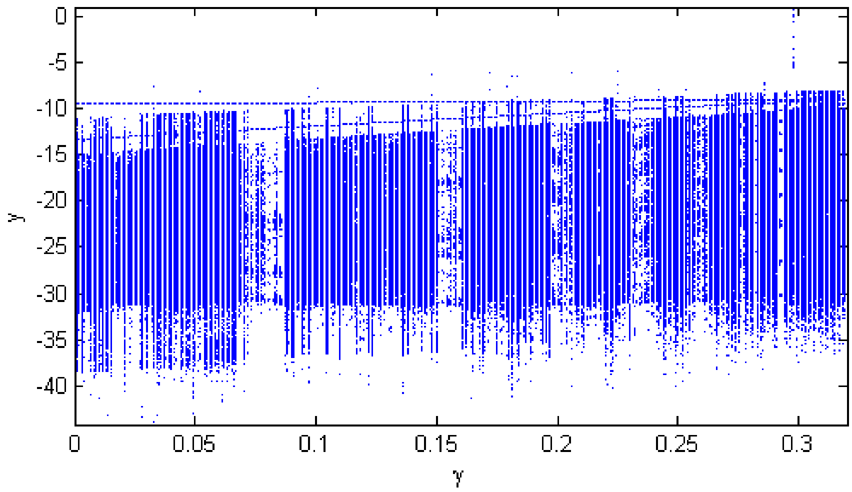

FIG. 5 BIFURCATION DIAGRAM VERSUS THE PARAMETER γ

Mappings are formed by these points as shownin Fig. 3.Fig.4 shows the Lyapunov exponents and bifuracation diagram of the system versus the parameter γ . Fig.5 shows bifuracation diagram of the system versus the parameter γ . From the Fig.4 and Fig.5, we can know when $\gamma \geq 0.3$, the system is hyper-chaotic.

3 CONCLUSIONS

In this paper，we proposes a new simple 4D memristive circuit and find the system .Through theoretical analysis, we find the system has abundant dynamic phenonmon. Such as chaos, hyper-chaos, etc. we confirme the existence of super-chaotic attractor by numerical simulations. Therefore, this article has a certain reference value for features of the memristor, and the study of chaotic and hyperchaotic system also has a positive meaning.

REFERENCES

[1] Leon. O. Chua. Memristor - missing circuit element, IEEE Transactions on circuit theory. 18 (1971) 507- 519.

[2] Dmitri B. Strukov, Gregory S. Snider, and Duncan R. Stewart, eds. The missing memristor found, Nature, 453 (2008) 80-83.

[3] Ricardo Riaza, Dynamical properties of electrical circuits with fully nonlinear memristors, Nonlinear Anal.-Real. 12 (2011) 3674-3686.

[4] Leon. O. Chua, Sung Mo Kang. Memristive devices and systems, Proceedings of the IEEE, 64 (1976) 209-223.

[5] Li. Z. Jun, Zeng. Y. Cheng, Li. Z. Bin. Memristive chaotic circuit based on modified SC-CNNs, Acta Physica Sinica, 63(2013) 010502.

[6] Xu. B. Rong. A simplest parallel chaotic system of memristor, Acta Physica Sinica, 62(2013)190506.

[7] Zhi. M. Luo, Asymptotic Behavior for Solution of Nonlinear Differential Equation, J.Comp. Appl.Math, 3(2014), 1-4.

[8] T. Driscoll, Y.V. Pershin, D.N. Basov, M. Di Ventra, Chaotic memristor, Appl. Phys. A-Mater. 102 (2011) 885-889.

[9] B. Muthuswamy, O. LEON, Simplest chaotic circuit, Int. J. Bifurcat. Chaos 20 (2010) 1567-1580.

[10] B.C. Bao, Z.H. Ma, J.P. Xu, Z. Liu, Q. Xu, A simple memristor chaotic circuit with complex dynamics, Int. J. Bifurcat. Chaos 21 (2011) 2629-2645.

[11] B. Muthuswamy, Implementing memristor based chaotic circuits, Int. J. Bifurcat. Chaos 20 (2010) 1335-1350.

[12] A. Buscarino, L. Fortuna, M. Frasca, L.V. Gambuzza, G. Sciuto, Memristive chaotic circuits based on cellular nonlinear networks, Int. J. Bifurcat. Chaos 22 (2012) 1250070.

[13] M. Itoh, L.O. Chua, Memristor Hamiltonian circuits, Int. J. Bifurcat. Chaos 21 (2011) 2395-2425.

[14] L. Chua, V. Sbitnev, H.S. Kim, Neurons are poised near the edge of chaos, Int. J. Bifurcat. Chaos 22 (2012) 1250098.

[15] A. Buscarino, L. Fortuna, M. Frasca, L.V. Gambuzza, A chaotic circuit based on Hewlett-Packard memristor, Chaos 22 (2012) 023136.