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Optimization and Empirical Analysis Based on Markov Forecasting Model

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Abstract

The optimization model based on Markov chain is established to optimize the prediction of industrial structure and provide reference for policy adjustment. The vectorization operator is used to transform the Markov prediction model into an optimization problem with constraints, which highlights the theoretical proof and computational rigor. Based on the data of three industrial structures in Yunnan Province from 1989 to 2019, this paper establishes Markov optimization model to predict the proportion of three industrial structures in Yunnan Province from 2020 to 2030. The maximum percentage average absolute error and hill inequality coefficient of the prediction are 1.2335% and 0.2, respectively. The order-degree of the three industrial structures is a stable series, which is stable around 1 after 1996. The sample data and the predicted values show four stages of change characteristics. After 2020, the three industrial structures are stable in the "three, two and one" structure.

Keywords: Markov Chain; Constrained Optimization; Three Industrial Structures in Yunnan Province; Degree of Order; Prediction

1 INTRODUCTION

The industrial structure of a region is related to the coordinated development of the overall local economy. The analysis of the local industrial structure reveals the advantages and deficiencies of its industrial structure. And through the guidance of active economic policies and measures, give full play to the role of competitive industries in economic development, gradually avoid and eliminate industries with not obvious advantages, realize the reasonable adjustment of industrial structure, and promote the rapid and efficient development of society and economy. The researchers have conducted a large number of qualitative and quantitative analysis of the industrial structure in a specific country or region from different perspectives and using different methods. The quantitative research mainly includes: (1) In articles of Peng[1], Wang[2] and Zhou[3], the industrial structure of Jiangxi, Hunan and Weinan City were studied by gray theory, and some methods and countermeasures were proposed. (2) In Zhao's[4-5] articles, By using Markov theory, the forestry industry structure of Guizhou and Guizhou was studied by solving the one-step transfer probability matrix. Li[6] analyzed the industrial structure of Yunnan Province with a method similar to Zhao's[4]. Hao[7] combined Markov theory with clustering method to analyze the regional industrial structure of Heilongjiang Province. The theory adopted in literature 4-7 is scientific and reasonable, but the solution process of the key parameters in the model is questionable and the rigor is not rigorous enough. In articles of Wang[8] and Bao[9], They used the Shift-Share analysis method to analyze the industrial structure of Panzhihua City and Shiyan City, Hubei Province respectively, and proposed the countermeasures to promote economic growth according to the actual situation of urban development. In articles of Xu[10] and Xie[11], they used the same method to analyze the forestry industrial structure of Fujian and Yunnan respectively. The essence of Shift-Share analysis is to apply the traditional static model to a specific region and specific industrial sector, which has certain limitations in reflecting the dynamic economic changes. There are more qualitative research materials about industrial structure than quantitative research, do not repeat.

On the basis of the Markov chain prediction model, this study transforms it into a constrained optimization problem by vectorization operator, and carries out some mathematical derivation. The program code is written in R language for strict calculation. The mathematical preciseness of the model solution is emphasized, the prediction accuracy of the model is guaranteed and the shortcomings of the above research are overcome. Combined with the order-degree, this paper makes an empirical analysis of the three industrial structures in Yunnan Province, and makes a prediction. The prediction results of the industrial structure in Yunnan Province in the next ten years can provide a reference for the formulation of local industrial policies. The software is Eviews and R, and PhotoShop is used to ensure that illustrations are intuitive.

2 MODEL ESTABLISHMENT AND SOLUTION

2.1 Markov Prediction Model and Optimization

Model assumption: (1) The industrial structure state of each time point is calculated at the time point of year, and the proportion of industrial output value three times a year remains unchanged. (2) The probability of state transition at each time point remains unchanged, that is, the proportion of output value of the three industries has the same trend at each time point. (3) The current industrial structure is only affected by the previous one.

Let the ratio of the j-Class industry to the GDP at the t-th time point is $y_j(t)$, $j = 1{:}m$, $t = 1{:}n$(year), taking it as a component, using Y_t^T=$(y_1(\mathrm{t}), \dots, y_m(\mathrm{t}))$to represent the proportion vector of m industries in t-th year (superscript denotes transpose, the same below), the state space contains m states. The vector Y_t^T is regarded as the estimation of the state probability at the t-th time point. the random variable $X_t = i$ indicates that the industry status at the t-th time point is the i-type industry, the conditional probability $p_{ij} = P(X_{t+1} = j|X_t = i)$ is the probability that state i will change to state j (event $X_{t+1} = j$occurrence) under the condition of given state i (event$X_t = i$occurrence), which is called one-step transition probability. And for arbitrarily i, there is $\sum_{j=1}^{m} p_{ij} = 1$. The data matrix and one-step transfer probability matrix of each industry from the first to n-th time points are recorded as: $Y_2 = \begin{bmatrix} Y_1^T \\ \dots \\ Y_n^T \end{bmatrix} \in R^{n\times m}$, $P = (p_{ij})_{n\times n} = \begin{bmatrix} p_{11} & \cdots & p_{1m} \\ \vdots & \ddots & \vdots \\ p_{m1} & \cdots & p_{mm} \end{bmatrix}$, Using $\mathrm{t} = 0$ to represent a base time point year, note: $Y_1 = \begin{bmatrix} Y_0^T \\ \dots \\ Y_{n-1}^T \end{bmatrix} \in R^{n\times m}$, The markov prediction model is:

$$Y_2 = Y_1 P + v, \text{ and } P\mathbf{1} = \mathbf{1} \tag{1}$$

Where bold 1 represents a vector of all components 1. For the convenience of calculation, the model (1) is transposed: $t(Y_2) = t(P)t(Y_1) + t(v)$.

Note: $X_2 = t(Y_2), Q = t(P), X_1 = t(Y_1), e = t(v)$ thus $X_2 = QX_1 + e$ (2)

After model (2) is processed by vectorization operator $vec(\cdot)$, it is recorded as:

$$vec(X_2) = (X_1^T \otimes I_m)vec(Q) + vec(e)$$

Note:$b = vec(X_2)$, $A = (X_1^T \otimes I_m) \in R^{nm\times \mathrm{m}^2}$, $x = vec(Q)$, $u = vec(e)$, $\otimes$ represents the Kronecker product of a matrix. The model is simplified as: $b = Ax + u$,The restrictive condition is:$(I_m \otimes \mathbf{1}^T)x = \mathbf{1}and0 \le \varepsilon_i^T x \le 1$, ε_i is the m^2-dimensional unit vector whose i-th component is 1 and the other components are 0. note $(I_m \otimes \mathbf{1}^T) = B$, thus, the model can be written as an optimization problem with constraints:

$$\begin{cases} \underset{x\in R^{m^2}}{\operatorname{argmin}} \|b - Ax\|_2^2, \ \text{s.t} \\ \qquad Bx = \mathbf{1} \\ 0 \le \varepsilon_i^T x \le 1 \quad i = 1{:}m^2 \end{cases} \text{ equivalent to } \begin{cases} \underset{x\in R^{m^2}}{\operatorname{argmin}} \frac{1}{2} x^t A^T Ax - b^T Ax, \ \text{s.t} \\ \qquad Bx = \mathbf{1} \\ 0 \le \varepsilon_i^T x \le 1 \quad i = 1{:}m^2 \end{cases} \tag{3}$$

Sets $V_1 = \{x: Bx = \mathbf{1}\}$ and $V_2 = \{x: 0 \le \varepsilon_i^T x \le \mathbf{1}\}$ are convex sets, respectively, and their intersection is also convex, the objective function $f(x) = \frac{1}{2} x^T A^T A x - b^T x, x \in R^{m^2}$ is convex, see appendix for proof. The constraint $Bx = \mathbf{1}$ is written as m equations, $B_{(j)}^T x - \mathbf{1} = 0$, $B_{(j)}^T$ is the j-th row vector of the matrix B, It represents m hyperplanes of R^{m^2} space. $0 \le \varepsilon_i^T x \le 1 \quad i = 1{:}m^2$ is the cube of R^{m^2} space.

For convenience, the equality and inequality constraints in optimization problem (3) are simplified, and the equality constraints are recorded as: $c_l(x) = 0,\ l = 1{:}m$, inequality constraints are recorded as: $c_l(x) = 0,\ l = (m+1){:}(2m^2)$. A simplified Lagrange function can be obtained as follows:

$$L(x,\lambda) = f(x) - \sum_{l=1}^{2m^2} \lambda_l c_l(x) \tag{4}$$

For(4), the gradient of with respect to x is calculated:

$$\nabla_x L(x,\lambda) = \nabla_x f(x) - \sum_{l=1}^{2m^2} \lambda_l \nabla_x c_l(x) \tag{5}$$

According to the Karush-Kuhn-Tucker theorem [13], if x^* is a possible local minimum of the constrained optimization problem (3), there must be a constant vector $\lambda^* = (\lambda_1^*, \dots, \lambda_{m^2}^*)$ such that (x^*, λ^*) is a KKT pair and satisfies the following conditions:

$$\lambda_l^* \ge 0,\ \lambda_l^* c_l(x^*) = 0, l = (m+1){:}2m^2 \tag{6}$$

The set of linearized feasible directions is as follows: $\mathcal{L}(x^*) = \{d \in R^{m^2}: \|d\| = 1, d^T \nabla c_i(x^*) = 0, i = 1{:}m, d^T \nabla c_i(x^*) \ge 0, i = (m+1){:}(m+r)\}$, r is the number of inequality constraints that work, $\mathcal{L}_1(x^*, \lambda^*) \subset \mathcal{L}(x^*)$, and it is the feasible direction set orthogonal to $\nabla f(x^*)$. The KKT point (x^*, λ^*) and the corresponding possible minimum point x^* are obtained by (5) and (6), for $\forall d \in \mathcal{L}_1(x^*, \lambda^*)$, because $d^T \nabla_{xx} L(x,\lambda) d > 0$, then x^* is a local minimum, according to the theorem in the appendix, x^* is the minimum point of the optimization problem (3). The concrete calculation process is explained in the empirical part, taking the industrial structure data of Yunnan Province as an example.

2.2 Order-degree

If the prediction results of models (1) and (3) are regarded as the target vector of industrial structure, the sample proportion of the three industrial structures is regarded as the concrete realization of the target vector, order-degree proposed by Dang [15] is used to reflect their proximity, the purpose is to estimate the similarity of the prediction results of the model after 2020.

$Y_t^T = (y_1(t), y_2(t), y_2(t))$ represents the proportion of the tertiary industrial structure at the t-th time point, $Y_{t0}^T = (y_{10}(t), y_{20}(t), y_{30}(t))$ is the target vector at the t-th time point, and their starting point zero vectors are $X_t^T = (0, x_2(t), x_3(t))$ and $X_{t0}^T = (0, x_{20}(t), x_{30}(t))$, the order degree of tertiary production structure is defined as:

$$r_{t0} = 1 - \frac{\sigma_t}{1 + \theta_0 + \theta_t} \tag{7}$$

Where $x_i(t) = y_i(t) - y_1(t)$, $x_{i0}(t) = y_{i0}(t) - y_{10}(t)$, $\theta_0 = |x_{20}(t)| + |x_{30}(t)|$, $\theta_t = |x_2(t)| + |x_3(t)|$, $\sigma_t = |x_2(t) - x_{20}(t)| + |x_3(t) - x_{30}(t)|$.

3 EMPIRICAL ANALYSIS

3.1 Solution and Error Analysis of Markov Optimization Model of Industrial Structure in Yunnan Province

The data of output value of three industries in Yunnan Province from 1989 to 2019 were collected（Among them, the three industrial data from 1989-2018 came from Yunnan Statistical Yearbook. The 2019 data are from the National Economic and Social Development Bulletin of Yunnan Province 2019）. Taking the proportion of 1989 as the basic value, using R language, the optimization problem (3) is solved by writing code through limSolve package [14], and the minimum value point is obtained. This value is transformed into the estimation of transition probability matrix by matrix operator.

$$\hat{P}=\begin{bmatrix}0.90706907 & 0.075240602 & 0.017690328\\ 0.03166660 & 0.958705981 & 0.009627419\\ 0 & 0 & 1\end{bmatrix}$$

The sum of squares of the absolute values of the residuals is 6.661338e-16, corresponding objective function value is $f(x^*)=\ 0.02415662$.

The results show that: in the next year (time point), transformation probabilities of the first industry to the second industry and the third industry in Yunnan Province are 0.0752406 and 0.01769033 respectively. Transformation probabilities from the second industry to the first and the third industry are 0.0316666 and 0.009627419 respectively. Transformation probabilities from the third industry to the first and second industry are 0 respectively. It fully shows that since 1990, the proportion of tertiary industry in Yunnan Province has gradually developed and expanded, and the proportion of first and second industries has gradually decreased.

Applying model (1) to forecast the proportion of three industrial structures in 1989-2019 (there is a lag, so the forecast value starts from 1990), the prediction results and the proportion of industrial structures are drawn together (see figure 1, in which the proportion sequence of the first, second and third industries are recorded as AG,IN and SE, each sequence is added after the corresponding mark F to indicate its prediction), Figure 1 shows that the proportion curves and prediction curves of the first, second and third industries are very close, with high prediction accuracy.

For back substitution and prediction, the percentage average absolute error, percentage average relative error and hill inequality coefficient (0 < Theil IC < 1) reflecting the difference between the fitting value and the true value of the proportion of the first, second and third industries are calculated respectively. The corresponding values are shown in Table 1. The maximum percentage average absolute error and the maximum percentage average relative error of the three industrial structure forecasts are 1.2335% and 3.4040%, respectively. The largest Hill inequality coefficient is 0.2, which is also relatively small, indicating that the prediction accuracy of model (1) and (3) is higher[16], the prediction results can truly reflect the change law of the three industrial structures in Yunnan Province, and can be used to predict the proportion of the three industrial structures in Yunnan Province in the future.

3.2 The Order-degree of Industrial Structure in Yunnan Province

Taking the three industrial structure predictions from models (1) and (3) as the target values and the sample proportion as the actual values, the order-degree series r_{t0} from 1990 to 2019 is calculated by formula (5). The sequence was tested by ADF [16]. Statistical values are less than the critical values of 1%, 5% and 10% (see Table 1). It shows that r_{t0} is a stationary sequence. Since 1996, the order-degree is greater than 0.981 (0.965 in 2019, slightly smaller, the data source in this year is different from that before, leading to a slightly larger prediction of the second and third industries), and the order-degree series is stable around 1. It shows that the actual sequence is very close to the target sequence (forecast), and it also shows that the industrial structure trend of Yunnan Province will be stable after 2019.

TABLE 1 EVALUATION INDEXES OF THE ACCURACY OF THE THREE INDUSTRIAL STRUCTURE PREDICTION MODELS OF YUNNAN PROVINCE FROM 1990 TO 2019

Precision index	The first industry	The second industry	The third industry
MAE（Average absolute error%）	0.7920	1.2335	1.1881
MAPE（Average relative error%）	3.4040	3.1012	3.1326
THeil IC(Hill inequality coefficient)	0.2000	0.1195	0.1108
Augmented Dickey-Fuller test statistic	t-Statistic:-7.830122	Prob.*:0.0000	
Test critical values:	1% level:-3.679322	5% level:-2.967767	10% evel:-2.622989

3.3 The Future Forecast of Industrial Structure in Yunnan Province

Through the model (1), we can get the forecast of the proportion of three industrial structures in Yunnan Province from 2020 to 2030 (see Figure 2).

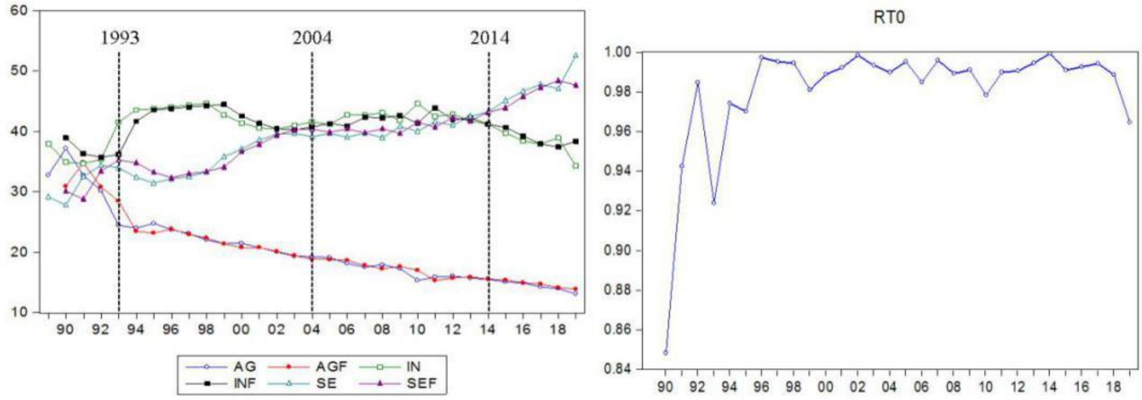

FIG. 1 THE GRAPH OF THE PROPORTION AND FORECASTED VALUE OF THE THREE INDUSTRIAL STRUCTURES IN YUNNAN PROVINCE FROM 1990 TO 2019(L) AND ORDERED-DEGREE SEQUENCE CURVE(R)

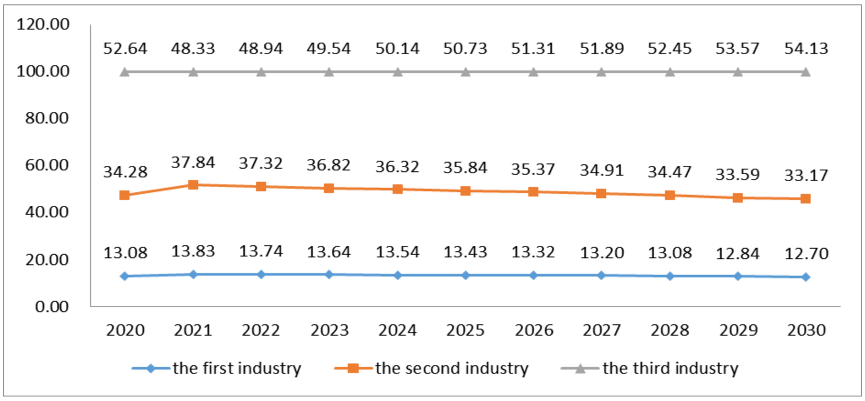

FIG. 2 THE FORECAST OF THE PROPORTION OF THREE INDUSTRIAL STRUCTURES IN YUNNAN PROVINCE FROM 2020 TO 2030

4 CONCLUSIONS AND RECOMMENDATIONS

The Markov prediction model is transformed into a constrained optimization problem by vectorization operator. The feasible region is a convex set and the corresponding objective function is a convex function, so the local minimum is the minimum. Empirical results show that: based on the output value data of the three industries in Yunnan Province from 1989 to 2019, the maximum average absolute error and average relative error of the three industrial structure forecasts are 1.2335% and 3.4040% respectively, and the error is small. The largest Hill inequality coefficient is 0.2, which is smaller. The ADF test shows that the prediction sequence of order- degree is stationary, and the mean value is 1. The autocovariance function only depends on the time interval and is independent of the time point. After 1996, the order-degree was more than 0.981 and was stable around 1. It fully shows that the Markov prediction optimization model has high prediction accuracy, and the prediction results of the three industrial structures of Yunnan Province in the future are reliable, which can provide the basis for the policy adjustment of the industrial structure of Yunnan Province.

Under the condition of model assumption, the results of one-step transfer probability matrix show that the first and second industries in Yunnan Province have the possibility of transferring to the third industry every year, but the third industry does not have the possibility of transferring to the first and second industries, and the third industry maintains sustainable development. From the perspective of industrial transfer probability, the transfer probability of the first industry to the second industry is greater than that of the second industry to the first industry, and the transfer probability of the first industry and the second industry to the third industry is 0.027317747.

The sample value and the predicted value of the proportion of industrial structure in Yunnan Province show four stages of change at the same time point (see Figure 1, left). In the first stage（before 1992, the characteristics of industrial structure were not obvious. In the second stage (1993-2003), the industrial structure of Yunnan Province was of "two, three and one" type. The second industry showed the change of decline after rise (convex upward) and the third industry showed the change of rise after decline (concave downward). The gap between the two was large, and the first industry showed a linear slow decline. In the third stage (2004-2013), the industrial structure of Yunnan Province is still of "two, three and one" type, but the proportion of the two industries is close to that of the three industries. The two industries rise steadily at the same time, and the first industry declines slowly at the same speed. In the fourth stage (2014-2019), the industrial structure of Yunnan Province is transformed into "three, two, one" type, It presents the three industrial structure types of economically developed regions. The third industry increased more rapidly than the second industry, and the second industry decreased rapidly. The first industry decreased slowly at the same speed. The forecast results after 2020 show that the industrial structure of Yunnan Province would have always maintained the "three, two and one" structure type, the three industries would have a steady upward trend, the second industry would has a steady downward trend, and the first industry would remain stable. In the next 10 years, the industrial scale of the three industries in Yunnan Province would exceed more than half of the total production scale.

Based on the above analysis results, suggestions are put forward to optimize the industrial structure of Yunnan Province: (1) the first industry and the third industry are highly integrated. To give full play to the role of modern service industry represented by electronic commerce, electronic information platform, financial service and new tourism industry, to support the first industry with information technology, and strive to improve the production efficiency of the first industry. (2) Give full play to the role of the second industry as a bridge. To introduce competition mechanism to inefficient traditional industrial enterprises and encourage innovation and development. To introduce talents, establish high-tech industrial parks, and encourage the development of high-tech industries from the policy and capital levels. (3) To give full play to the regional advantages and focus on the development of the tertiary industry. We should make full use of Yunnan's advantages in tourism resources and ethnic cultural diversity, strive to cultivate new tourism formats, and plan a number of service industry clusters represented by modern service industry with the help of the influence of central Yunnan urban economic circle, so as to lay a solid foundation for the future development of modern service industry.

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APPENDIX: THE PROOF OF LEMMA AND THEOREM

Lemma 1. Let two subsets in R^{m^2},$V_1 = \{x: Bx = \mathbf{1}\}$ and $V_2 = \{x: 0 \le \varepsilon_i^T x \le \mathbf{1}\}$ then: (1) $V_1$and V_1 are convex sets. (2)$V_1 \cap V_2$ is convex set.

Proof. **(1)** when $m \ge 2$,$rank(B) = rank(I_m \otimes \mathbf{1}^T) = m \le m^2$, Easy to verify $rank(B) = rank(B, \mathbf{1})$ and $\eta \triangleq \mathbf{1} \otimes \varepsilon_1$ is a solution of equation $Bx = \mathbf{1}$. Let $\xi^T = (\xi_1, \dots., \xi_{m^2-m})$ is the base of the solution space of equation $Bx = \mathbf{0}$, c is a constant vector in R^{m^2}, according to the knowledge of linear algebra, V_1 can be expressed as $V_1 = \{x; x = \eta + \xi^T c\}$.

Let arbitrary $0 < \alpha < 1, x, y \in V_1$, then there are two m^2dimensional constant vectors c_1, c_2 such that $x = \eta + \xi^T c_1$ and $y = \eta + \xi^T c_2$ are the solutions of equation $Bx = \mathbf{1}$ respectively, because of $B(\alpha x + (1 - \alpha)y) = \mathbf{1}$,$\alpha x + (1 - \alpha)y \in V_1$, so V_1 is convex set.

Let arbitrary $0 < \beta < 1, u, v \in V_2$, then $0 \le \beta \varepsilon_i^T u \le 1, 0 \le (1 - \beta)\varepsilon_i^T v \le 1$, such that $0 \le \varepsilon_i^T(\beta u + (1 - \beta)v) \le 1, \beta u + (1 - \beta)v \in V_2$, so V_2 is convex set.

(2) Let arbitrary $0 < \gamma < 1, l, m \in V_1 \cap V_2$, because of $l, m \in V_1, V_1$ and V_1 are convex sets, then $\gamma l + (1 - \gamma)m \in V_1$. The same reason, because of $l, m \in V_2, V_2$ is convex set, then $\gamma l + (1 - \gamma)m \in V_2$, $\gamma l + (1 - \gamma)m \in V_1 \cap V_2$, so $V_1 \cap V_2$ is convex set.

Lemma 2. The objective function $f(x) = \frac{1}{2}x^T A^T A x - b^T x, x \in R^{m^2}$is convex function.

Proof. To calculate the gradient for $f(x)$, then $\nabla f(x) = A^T A - b$,$\nabla^2 f(x) = A^T A$. Let arbitrary $x \in R^{m^2}$and $x \ne 0$ then $x^T A^T A x = \|Ax\|_2^2 \ge 0$, so $f(x)$ is convex function.

Theorem. The objective function $f(x) = \frac{1}{2}x^T A^T A x - b^T x, x \in R^{m^2}$. Under the constraints of model (3) whose local minimum is the global minimum.

Proof. According to lemma 1 and lemma 2, the feasible region of optimization problem (3) is as follows:$D = \{x \in R^{m^2}: Bx = \mathbf{1};\ 0 \le \varepsilon_i^T x \le 1 \quad i = 1: m^2\}$=$V_1 \cap V_2$, it is convex set. the objective function $f(x)$ is convex function. Let x^* be its minimum, from the property of convex function on convex set [12], we can know that a is a global minimum of the objective function.