Journal of Optics Applications
December 2015, Volume 4, Issue 2, PP.7-14

Characteristics of Surface Plasmon Polaritons with Effective Permittivity

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Abstract

Effective permittivity is a propagation constant of surface plasmon polaritons, which help visualise how surface plasmon excitation travels in space. Compared with the traditional method with frequency parameters, this method is more intuitive and practical.

Keywords: Surface Plasmon Polaritons; Maxwell's Equations; Simulation

1 INTRODUCTION

A surface plasmon polaritons (SPPs) is an electromagnetic excitation existing between the surface of metal and dielectric. It is an intrinsically two-dimensional excitation whose electromagnetic field decays exponentially with distance from the surface.

In this paper, the principle of surface plasmon excitation has been introduced. Under single flat interface and metal / dielectric multilayer structure system, transverse electric and magnetic wave modes have been derived from the Maxwell equations. From the analyses, with software Mathematica, propagation of surface plasmons has been simulated, analyse effective permittivity varies with of a variety of metal /dielectric and wavelength in a single plane of the interface, and the changes of effective permittivity in the case with the thickness of the multilayer heterostructure and the wavelength changes. Simulation results have been compared with the theoretical value. A method as using the effective dielectric constant of the surface plasmons propagation has been proposed.

2 BASIC CHARACTERISTICS OF SPPS

To analyse the physical properties of SPPs, a classical model of SPPs is illustrated in FIG. 1.

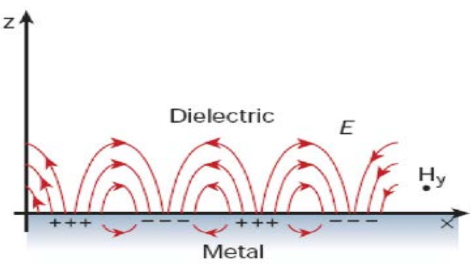

FIG. 1 CLASSICAL MODE OF SURFACE PLASMON POLARITONS

Apply Maxwell’s equations to the flat interface between dielectric and metal. The electromagnetic equation is derived in the general form firstly.

Use the curl equation for the electric field.

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{1}$$

$$\nabla \times \nabla \times E = -\mu_0 \frac{\partial^2 D}{\partial t^2} \quad (2)$$

According to the external charge (ρ_{ext}) and the current densities (J_{ext}) are absent.

$$\nabla \times D = J_{ext} + \frac{\partial D}{\partial t} \quad (3)$$

$$D = \varepsilon_0 E + P \quad (4)$$

P describes the electric dipole moment per unit volume inside the material, caused by the alignment of microscopic dipoles with the electric field. P is zero in this situation.

Consider that there are no external stimuli and expand the formula.

$$\nabla \cdot D = 0 \quad (5)$$

As the variation of the dielectric profile can be neglect, $\varepsilon = \varepsilon(r)$.

$$\nabla^2 E - \frac{\varepsilon}{c^2}\frac{\partial^2 E}{\partial t^2} = 0 \qquad (\mu_0 \varepsilon_0 = \frac{1}{c^2}) \quad (6)$$

Consider the harmonic time dependence (E related to time and position) of the electric field.

So Helmholtz equation,

$$\nabla^2 E + k_0^2 \varepsilon \frac{\partial^2 E}{\partial t^2} = 0 \quad (k_0 = \frac{\omega}{c}) \quad (7)$$

Define the propagation geometry in FIG.2. Let ε depends only on one spatial coordinate, so $\varepsilon = \varepsilon(z)$, the waves propagate along the x-direction and show no spatial variation in the perpendicular of a Cartesian coordinate system.

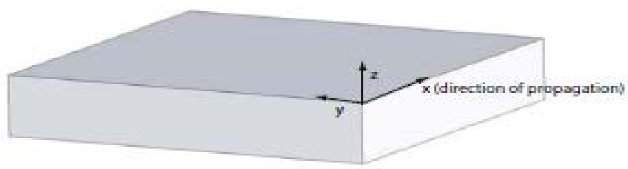

FIG. 2 DEFINE THE PROPAGATION GEOMETRY IN CARTESIAN COORDINATE SYSTEM

Applied to electromagnetic surface, on the plane $z = 0$, the traveling waves equation is

$$E(x, y, z) = E(z)e^{-i\beta x} \quad (\beta = kx) \quad (8)$$

β is the propagation constant of the propagating waves and relates to the constituent of the wave vector in the direction of propagation. The propagation direction is illustrated in FIG.3.

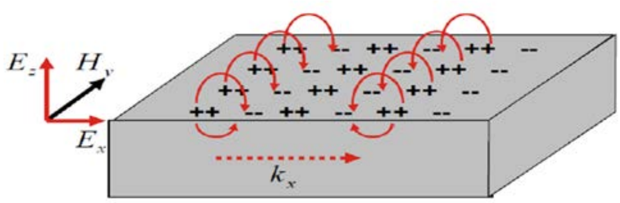

FIG. 3 THE PROPAGATION COMPONENTS OF SPPS

Rewrite the Helmholtz equation

$$\frac{\partial^2 E(z)}{\partial t^2} + (k_0^2 \varepsilon - \beta^2) E = 0 \quad (9)$$

The equation for the magnetic field H is similarly. The propagation direction is illustrated in FIG.4.

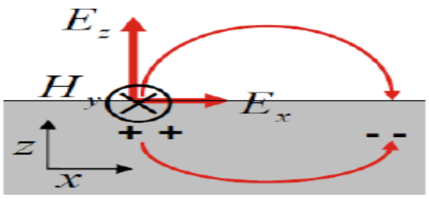

FIG. 4 THE DYRECTION OF ELECTRONMAGNETISM FOR SPPS

From the equation (9), we can make the general analysis of guided electromagnetic modes in waveguides, even extend to its properties and applications.

For determining the characteristics of spatial field profile and dispersion of propagating waves. The explicit expressions for the different field components of E and H can be achieved by the curl equation (1).

As the propagation along the x-direction, and is homogenies in the y-direction.

Simplifies the system of equation

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = i\omega\mu_0 H_x \tag{10a}$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = i\omega\mu_0 H_y \tag{10b}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = i\omega\mu_0 H_z \tag{10c}$$

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = -i\omega\varepsilon_0\varepsilon E_x \tag{10d}$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = -i\omega\varepsilon_0\varepsilon E_y \tag{10e}$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = -i\omega\varepsilon_0\varepsilon E_z \tag{10f}$$

There exist two sets of self-consistent presentation in this system with different polarization properties of the propagating waves.

In transverse electric modes, the components of field, E_x, E_z and H_y are nonzero.

$$E_x = -i\frac{1}{\omega\varepsilon_0\varepsilon}\frac{\partial H_y}{\partial z} \tag{11a}$$

$$E_z = -\frac{\beta}{\omega\varepsilon_0\varepsilon} H_y \tag{11b}$$

The wave equation for transverse electric mode become

$$\frac{\partial^2 H_y}{\partial z^2} + (k_0^2\varepsilon - \beta^2) H_y = 0 \tag{12}$$

In the set of the transverse magnetic modes, H_x, H_z and E_y being nonzero.

$$H_x = -i\frac{1}{\omega\mu_0}\frac{\partial E_y}{\partial z} \tag{13a}$$

$$H_z = \frac{\beta}{\omega\mu_0} E_y \tag{13b}$$

The wave equation for transverse magneti mode become

$$\frac{\partial^2 E_y}{\partial z^2} + (k_0^2 \varepsilon - \beta^2) E_y = 0 \tag{14}$$

Consider with the most simple geometry sustaining SPPs as a single, flat interface between a dielectric, non-absorbing half space ($z > 0$) with positive real dielectric constant and an adjacent conducting half space ($z < 0$) described via a dielectric function $\varepsilon_1(\omega)$.

For transverse magnetic mode, in both half spaces yields separately

For $z > 0$

$$\mathrm{H_y}(z) = A_2 e^{i\beta x} e^{k_2 z} \tag{15a}$$

$$E_x(z) = iA_2 \frac{1}{\omega \varepsilon_0 \varepsilon_2} e^{i\beta x} e^{-k_2 z} \tag{15b}$$

$$E_z(z) = -A_1 \frac{1}{\omega \varepsilon_0 \varepsilon_2} e^{i\beta x} e^{-k_2 z} \tag{15c}$$

For $z < 0$

$$H_y(z) = A_1 e^{i\beta x} e^{k_1 z} \tag{15d}$$

$$E_x(z) = -iA_1 \frac{1}{\omega \varepsilon_0 \varepsilon_1} e^{i\beta x} e^{k_1 z} \tag{15e}$$

$$E_z(z) = -A_1 \frac{\beta}{\omega \varepsilon_0 \varepsilon_1} e^{i\beta x} e^{k_1 z} \tag{15f}$$

k_i is the component of the wave vector perpendicular to the interface in the two media $k_i \equiv k_{zi} (i = 1, 2)$ .With evanescent decay in the perpendicular z-direction, the reciprocal value, $\hat{z} = 1/|k_z|$. $\hat{z}$ is the evanescent decay length of the fields perpendicular to the interface which quantifies the confinement of the wave.

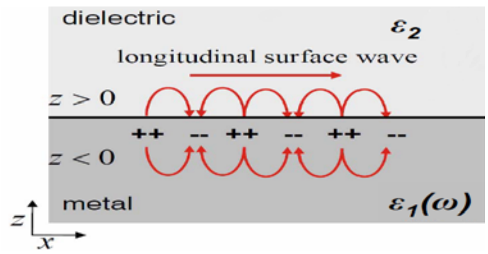

FIG. 5 THE PROPGRATION OF ONE-FLAT SPPS

According to the continuity of H_y and $\varepsilon_i E_z$ at the interface.

$$A_1 = A_2 \tag{16a}$$

$$\frac{k_1}{k_2} = -\frac{\varepsilon_1}{\varepsilon_2} \tag{16b}$$

The expression for H_y further has to fulfil the wave equation.

$$k_1^2 = \beta^2 - k_0^2 \varepsilon_1 \tag{17a}$$

$$k_2^2 = \beta^2 - k_0^2 \varepsilon_2 \tag{17b}$$

The central result represent the characteristics of SPPs at the interface between the two half spaces.

$$\beta = k_0 \sqrt{\frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2}} = k_0 n_{eff} \tag{18}$$

Effective permittivity ε_{effect} represent characteristics of SPPs.

$$n_{eff} = \sqrt{\frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2}} \tag{19}$$

As in transverse magnetic mode, the change of dielectric function $\varepsilon_1(\omega)$ at metal interface, so we can use the Drude dielectric function.

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma\omega} \tag{20}$$

Set the dielectric material as air ($\varepsilon_2 = 1$).

According to the metal dielectric constant and the wavelength data, we can use Mathematics to calculate the effective permittivity. The relationship of effective permittivity and wavelength is been plot in Fig 6.

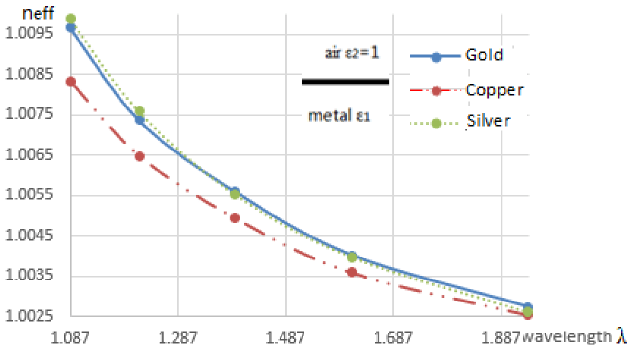

FIG. 6 THE EFFECTIVE PERMITTIVITY OF ONE-FLAT SPPS

3 SPPS IN MULTILAYER SYSTEMS

Consider the SPPs in multilayer systems, each single interface of conducting and dielectric thin films can sustain boundary situation. As the separation between the same interfaces is equal or smaller than the decay length $\hat{z}$ of the interface mode, interactions between SPPs induce coupled modes. First one, a thin metallic layer sandwiched between two infinite thick dielectric claddings, which is an insulator/metal/insulator (IMI) heterostructure. And secondly a thin dielectric core layer (I) sandwiched between two metallic claddings (II, III), which is a metal/insulator/metal (MIM) heterostructure.

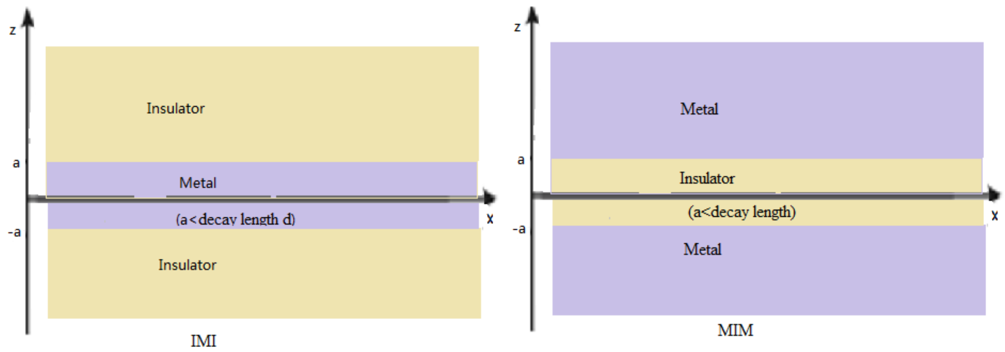

FIG. 7 THE MULTILAYERS SYSTEM OF SPPS

For the lowest-order bound modes, a general description of transverse magnetic modes has no oscillation in the Z-direction, the propagation direction perpendicular to the interfaces.

Using the system of governing equations for transverse magnetic modes, we obtain the wave components of the field.

For $z > 0$

$$H_y = Ae^{i\beta x} e^{-k_3 z} \tag{21a}$$

$$E_x = iA \frac{1}{\omega \varepsilon_0 \varepsilon_3} k_3 e^{i\beta x} e^{-k_3 z} \tag{21b}$$

$$E_z = -A\frac{\beta}{\omega\varepsilon_0\varepsilon_3}k_3 e^{i\beta x}e^{-k_3 z} \tag{21c}$$

For $z < 0$

$$H_y = Be^{i\beta x}e^{k_2 z} \tag{21d}$$

$$E_x = -iB\frac{1}{\omega\varepsilon_0\varepsilon_2}k_2 e^{i\beta x}e^{k_2 z} \tag{21e}$$

$$E_z = -B\frac{\beta}{\omega\varepsilon_0\varepsilon_2}k_3 e^{i\beta x}e^{k_2 z} \tag{21f}$$

In order to simplify the discussion, the component of the wave vector perpendicular to the interfaces is sampled as $k_i \equiv k_{zi}$. In the sandwiched region ($-a < z < a$), the modes localized at the bottom and top interface couple. Since the requirement of continuity of H_y and E_x, we get the fellow relationship, a linear system of four coupled equations. The equation of H_y further fulfill relationship between the three distinct regions.

Using the system of linear equations to solve the dispersion relation between β and ω, the results in an implicit expression.

$$e^{-4k_1 z} = -\frac{k_1/\varepsilon_1 + k_2/\varepsilon_2}{k_1/\varepsilon_1 - k_2/\varepsilon_2}\frac{k_1/\varepsilon_1 + k_3/\varepsilon_3}{k_1/\varepsilon_1 - k_3/\varepsilon_3} \tag{22}$$

Consider about a special condition that the dielectric response of the substructure (cladding II) and the superstructure (cladding III) are equally.

In this special case, the dispersion relation is split into two different equations for odd and even modes.

The modes of odd vector parity, as $E_x(z)$ is odd, $H_y(z)$ and $E_z(z)$ are even functions

$$\tanh k_1 a = -\frac{k_2\varepsilon_1}{k_1\varepsilon_2} \tag{23a}$$

The modes of odd vector parity, as $E_x(z)$ is even, $H_y(z)$ and $E_z(z)$ are odd functions

$$\tanh k_1 a = -\frac{k_1\varepsilon_2}{k_2\varepsilon_1} \tag{23b}$$

The pair of equations can be applied to IMI and MIM heterostructure. We investigate the properties of the coupled SPP modes in IMI geometry - a thin metallic film of thickness 2a sandwiched between two insulating layers.

As shown in Fig8, ε_1 is the dielectric function of the metal, $\varepsilon_1 = \varepsilon_1(\omega)$, ε_2 is the real dielectric constant of the dielectric cladding in substructure and the superstructure.

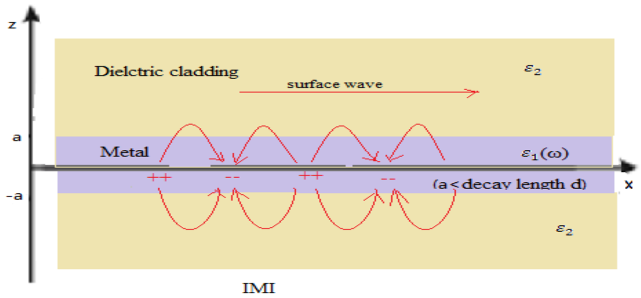

FIG. 8 THE SYMMETRICAL PROPATION MODE IN IMI MULTILAYER SYSTEM OF SPPS

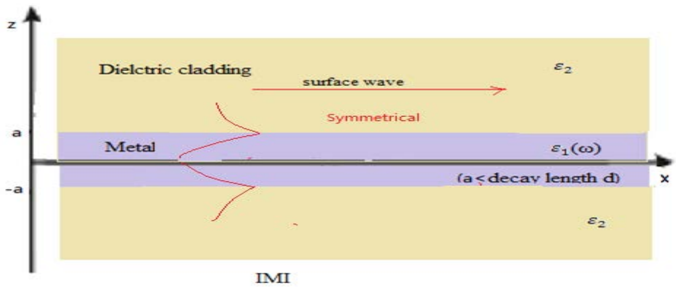

FIG. 9 THE ASYMMETRICAL PROPATION MODE IN IMI MULTILAYER SYSTEM OF SPPS

Newton's method is been used to solve equations.

$$x_{n+1} = x_n - \frac{f(x)}{f'(x)} \tag{24}$$

We use Mathematica to simulation the relationship between n_{eff} and thickness d.

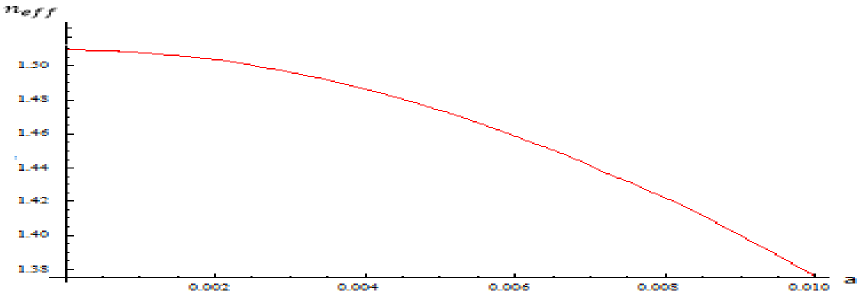

FIG. 10 THE EFFECTIVE CONSTANT OF COPPE

We can find that when the separation a increase, n_{eff} will decrease in the same time. When the separation is relative small with the decay length, it almost have no effect on the effective permittivity. As the separation being very large, the effective permittivity is jump seriously.

Then, we fixed the separation a as 500 nm and change the dielectric constant of the metal and dielectric ε_2 and $\varepsilon_1(\omega)$ for different wavelength.

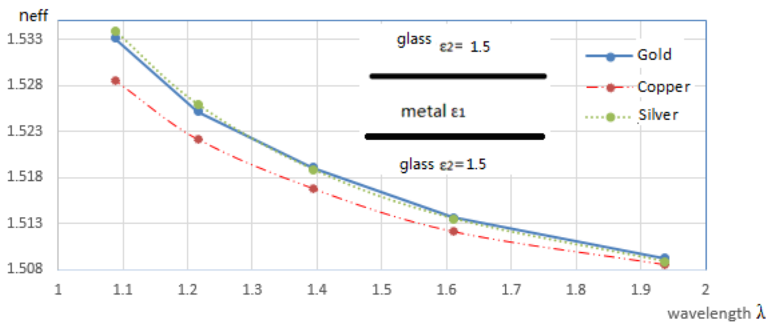

FIG. 11 THE EFFECTIVE CONSTANT OF GLASS AND METAL

The graphics are concave curve. When the wavelength increases, the permittivity of SPPs decreases. The maximum value is close to the dielectric constant. All of the plots describe the situation of odd mode. When the propagation mode is even, the curve upward bending.

4 CONCLUSIONS

The parameter of effective permittivity is visualise how surface plasmon excitation travels in space. With the rapid development of optical techniques, control and manipulation of light using SPPs on the nanometre scale exhibit significant advantages in nanophotonics devices with very small elements, and SPPs open a promising way in fields involving environment, energy, biology, even medicine. Compared with the traditional method with frequency parameters, effective permittivity provide a more intuitive observation of SPPs. In practice, controlled the system with adjust the effective permittivity can reduce the conversion process reach the anticipated result.

ACKNOWLEDGMENT

Appreciate for the support by the International Conference on Photonics and Optoelectronics.

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