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Nonlinear Dynamics for a New Chen-Lee-like Chaotic System

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Abstract

A new autonomous chaotic system obtained from the Chen-Lee chaos system is studied. The nonlinear dynamic and the existence of attractor of the new Chen-Lee-like autonomous system is analyzed by bifurcation diagrams, Lyapunov-exponent, Poincaré section and phase portraits etc. The research shows that the new four dimensional systems differs from Chen-Lee system completely, and presents some distinct nonlinear properties.

Keywords: New Chen-Lee-like Chaos; Lyapunov Ddimension; Bifurcation Diagrams; Chaotic System

INTRODUCTION

Chaos, widely observed in nonlinear dynamic systems, is one of the most important scientific discoveries in the last century. In 1963, the first chaotic system in a three autonomous ordinary differential equations was founded by Lorenz. With development of non-linear dynamics recently, a variety of chaotic attractor has also been proposed and has been studyed widely [1-5].

Recently some researchers have studied fractional order calculus theory. Some novel chaotic systems with fractional order , such as the Rössler system with fractional order [6], the Chen system with fractional order [7], Liu system with fractional order [8], the Lü system with fractional order [9], the generalized augmented Lü system with fractional order [10], etc [11-15], are successfully investigated. Numerical methods for simulating fractional order chaotic system are proposed.

In this article r, a new four-dimensional chaotic attractor is obtained on account of the Chen-Lee chaos[16]. The nonlinear characteristic of this new chaotic system is illustrated from bifurcation diagrams, Lyapunov-exponent, Poincarmap and phase trajectory, etc.

DYNAMICAL CHARACTERISTICS OF A NOVEL CHEN-LEE-LIKE CHAOTIC ATTRACTOR

A novel four-dimensional chaotic attractor based on the Chen-Lee chaos is obtained. The nonlinear differential equations is constructed from

$$\begin{cases} \dot{x} = ax - yz \\ \dot{y} = xz - cy \\ \dot{z} = xy - bz - w \\ \dot{w} = xyz - xz - dw \end{cases} \tag{1}$$

where $\boldsymbol{x} = (x, y, z, w)^T \in \boldsymbol{R}^4$ are the state variables of the differential equations (1), a, b, c, d are positive constant coefficients .Equations (1) contains five non-linear items, which has a strong non-linear characteristics.

SYMMETRICAL CHARACTERISTIC AND INVARIANCE PROPERTY

Firstly, observe that the Lorenz equations has a symmetry S , on account of the transformation

$S:(x, y, z) \to (-x, -y, z)$, which allows Lorenz equation is invariance property for all values of the parameters.

Clearly, the new system (1) has no symmetry.

DISSIPATIVITY AND THE CHAOTIC ATTRACTOR

At the same time, the equations have dissipativity, on account that the divergence of the vector field (2), as well known as the trajectory (4) of the Jacobian matrix (3), is negative:

$$\nabla V = \frac{1}{V}\frac{\mathrm{d}\vec{V}}{\mathrm{d}t} = \mathrm{div}\vec{V} = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{w}}{\partial w} = Tr(\boldsymbol{J}) = \sum_{i=1}^{4}\lambda_i = \sum LES \tag{2}$$

$$\boldsymbol{J} = \begin{bmatrix} a & -z & -y & 0 \\ z & -c & x & 0 \\ y & x & -b & -1 \\ -z+yz & xz & xy-x & -d \end{bmatrix} \tag{3}$$

$$Tr(\boldsymbol{J}) = a - c - b - d \tag{4}$$

where $\lambda_i (i = 1, 2, 3, 4)$ are the characteristic roots of the Jacobian matrix (3), LEs express the four Lyapunov exponents of the equation(1).

When and only when $a - b - d - c > 0$, the system(1) will always be dissipative. This is, the volume element $V(0)$ is reduced by the inflow to a volume element $V(0)e^{(a-b-d-c)t}$ as time $t \to \infty$, Those show that every volume containing the equations trace reduces to zero as $t \to \infty$ by an exponential ratio $a - b - d - c$. Consequently, all equation traces are finally restricted to a specific subset with zero volume and the asymptotic movement settles onto the novel chaotic attractor. These indicate that the kinetics will be inclined to an chaotic attractor as $t \to \infty$.

LYAPUNOV EXPONENT AND THE LYAPUNOV DIMENSION

When $a = 5, b = 3, c = 10, d = 38$, numerical simulation illustrates that equation (1) has the below Lyapunov exponents: $\lambda_{L1} = 0.8840, \lambda_{L2} = 0.0213, \lambda_{L3} = -9.3300, \lambda_{L4} = -39.0456.$ The system(1) having two positive Lyapunov exponents is hyperchaotic according to chaos theory.

Let initial condition $(-0.1, 0.2, -0.5, 0.3)$, a hyperchaotic attractor is obtained as seen in Fig. 1.

To be a clear view of the chaotic attractor, Fig. 2 illustrates the projections of the chaotic attractor on corresponding planes of coordinates.

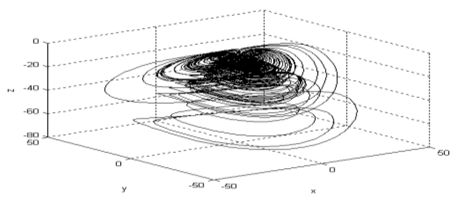
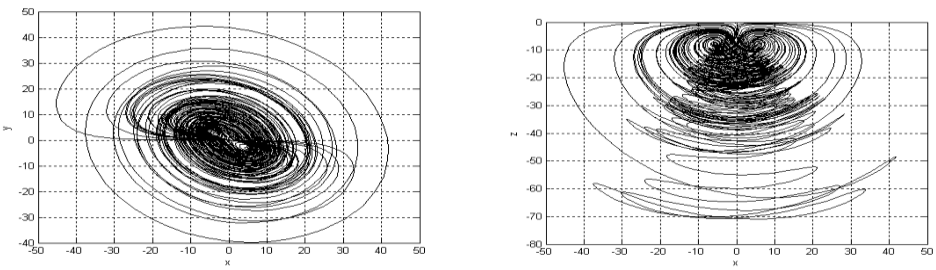


FIG.1. THE CHAOTIC ATTRACTOR OF EQUATION (1) WITH COEFFICIENT VALUES A = 5, B = 3, C = 10, D = 38,



(a) (b)

FIG. 2. SIMULATED PHASE TRAJECTORYS OF THE EQUATION (1) WITH COEFFICIENT VALUES A = 5, B = 3, C = 10, D = 38, PROJECTED ON(A) THE X–Y PLANE; (B) THE X–Z PLANE;

According to Kaplan-Yorke conjecture formula, Lyapunov dimension is obtained

$$
\begin{aligned}
D_L &= k + \frac{1}{\left|\lambda_{L,k+1}\right|} \sum_{i=1}^{k} \lambda_{L,i} = 2 + \frac{\lambda_{L1} + \lambda_{L2}}{\left|\lambda_{L3}\right|} \\
&= 2 + \frac{0.8840 + 0.0213}{\left|-9.3300\right|} = 2.0970
\end{aligned}
\tag{10}
$$

From Figure 3, the largest Lyapunov exponents of equation (1) are bigger than zero, and the system's Lyapunov dimension is not an integer, thus we confirm that the equation (1) is hyperchaotic.

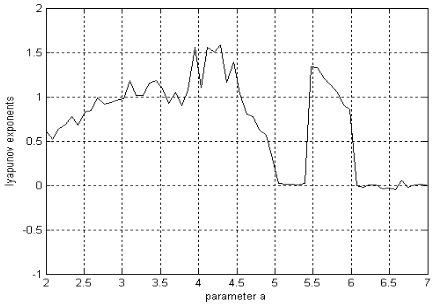


FIG.3. THE LARGEST LYAPUNOV EXPONENT SPECTRA OF SYSTEM (1)

POINCARÉ MAP AND BIFURCATION ANALYSIS

The Poincaré sections and the corresponding bifurcation phase figure of equation variable are very useful tools to observe system (1). Figure 4 indicates the Poincaré images on $y-z(x=0)$ plane .From Figure. 4, a lot of dense points with fractal structure, which led to the complex dynamic behavior of the system (1), are founded. While the coefficients $b=3, c=26, d=38$, ' a ' is valued in the closed interval [4, 7]. The bifurcation diagram of parameter a is showed from Fig. 5

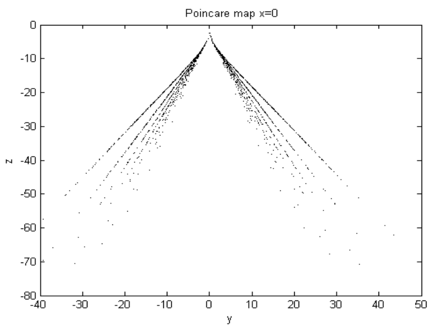


FIG.4. THE POINCARÉ IMAGE ON THE $y-z(x=0)$ PLANE

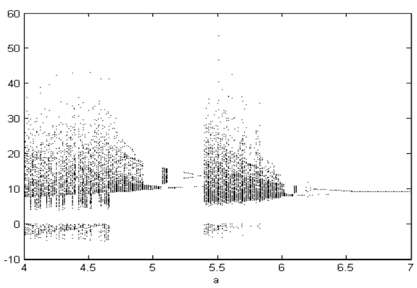


FIG.5 BIFURCATION DIAGRAM VERSUS THE CONTROL PARAMETER a

CONCLUSIONS

In this Letter, a novel Chen-Lee like hyperchaotic equation is constructed. The dynamic characteristic and nonlinear properties of the four-dimensional autonomous equations are researched.

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