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The Reliability of a Repairable System with Reserve Components

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Abstract

In this paper, a kind of human-machine system with general failed system repair time distribution is studied. By using the theory of the linear operator in semi-groups, we proved that the non-negative time-dependent existence and uniqueness, asymptotic stability and exponential stability of the solution. At last, we show a computer numerical simulation of the steady-state availability of warning system and non-warning system by mathematical software.

Keywords: Human-machine System; Stability; Steady-state Availability

1 INTRODUCTION

The repairable system consists of two running components, both units start operating instantaneously with the warm standby and its switching mechanism in perfect condition. The standby switching mechanism which comes on automatically in the event of a failure can also fail but it is repairable. The system can fail due to a common-cause failure from the normal operating condition as well as due to hardware failures. By using the theory of Semi-groups of linear operators in Banach space. We have proved the existence and uniqueness of solution, semi-discretization, and asymptotic stability. The spectral characteristic index and stability of the system, and its reliability is analyzed.

According to [1], the state diagram of the system is shown in Figure 1:

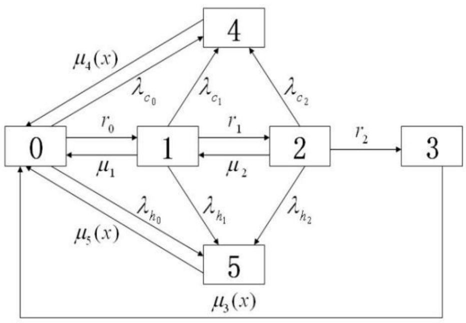

FIGURE 1

The following assumptions are associated with this system:

- The system has three identical units (two active and one standby).
- The hardware and common-cause failures are statistically independent.
- A common-cause failure will occur when both active parallel units and the standby are in perfect working condition as well as, when the system is operating in a degraded state.
- The system is in an up-state as long as one unit is working.
- All failure rates are constant.

- The failed system repair times are assumed to be arbitrarily distributed.
- A repaired unit or system is assumed to be as good as new.
- The switching mechanism for the standby is considered automatic and instantaneous.
- The standby may fail in its standby mode, in addition, to the switching mechanism.

Where i=0 said two unit and a standby parts are in good condition; i=1 represents a component of operation by hardware error fault, a standby unit immediately add operation; i=2 said only a part of the normal operation of the system without reserve components; i=3 system in fault state due to hardware errors; i=4 that system is in the fault state due to the usual error; i=5 system in fault state due to critical human error; i=c system in fault state due to the usual error; but the system is still running.

The model can be described by Integra differential equations as follows:

$$\begin{cases} \dfrac{dp_0(t)}{dt} = -a_0 p_0(t) + \mu_1 p_1(t) + \sum_{j=3}^{5} \int_0^{\infty} p_k(x,t)\mu_k(x)dx, \\ \dfrac{dp_1(t)}{dt} = r_0 p_0(t) - a_1 p_1(t) + \mu_2 p_2(t), \\ \dfrac{dp_2(t)}{dt} = r_1 p_1(t) - a_2 p_2(t), \\ \dfrac{dp_c(t)}{dt} = -\lambda_c p_c(t) + \lambda_{c_0} p_0(t), \\ \dfrac{\partial p_k(x,t)}{\partial t} + \dfrac{\partial p_k(x,t)}{\partial x} = -\mu_k(x) p_k(x,t), k = 3,4,5 \end{cases} \tag{1.1}$$

Where: $a_0 = r_0 + \lambda_{c_0} + \lambda_{h_0}, a_1 = r_1 + \lambda_{c_1} + \lambda_{h_1} + \mu_1, a_2 = r_2 + \lambda_{c_2} + \lambda_{h_2} + \mu_2$

$p_3(0,t) = r_2 p_2(t)$, $p_1(0,t) = \sum_{i=0}^{2} \lambda_{c_1} p_i(t)$, $p_5(0,t) = \sum_{i=0}^{2} \lambda_{h_1} p_i(t) + \lambda_c p_c(t)$

When $t=0$, $p_0(0)=1$, $p_1(0) = p_2(0) = p_c(0) = 0$, $p_3(x,0) = p_1(x,0) = p_5(x,0) = 0$.

i,k : the number of states in which the system is located;

r_i :The system is in a state of constant hardware failure rate of i(i=0,1,2);

μ_i :The system is in constant state, and i has constant hardware recovery(i=1.2);

$\mu_k(x)$:The system is in state j and the repair rate is x of the repair time(k=3,4,5);

λ_c :Steady state failure rate of the system from State c to state 4;

λ_{c_i} :Steady state failure rate of the system from State i to state 4(i=0,1,2);

λ_{h_i} :The constant critical human failure rate of the system from state i to state 5(i=0,1,2);

$p_i(t)$:The probability that the moment t is in the state i(i=0,1,2,c);

$p_k(x,t)$:The system is at the moment t in the state k, and the repair time is x(t>0,k=3,4,5).

We suppose that for all $T > 0$,

$\int_0^T \mu_k(\xi)dx < \infty$, $\int_0^\infty \mu_k(\xi)d\xi = \infty$, $\int_0^\infty e^{-\int_0^x \mu_k(\xi)d\xi}dx < +\infty$, $k = 3,4,5$, $0 < c_k < \lim_{x\to\infty}\frac{1}{x}\int_0^x \mu_k(\xi)d\xi$, $0 \le \mu_k(x) < \infty$

Choose the state space as $X = C^5 \times (L^1(R^+))^3$,

It is obvious that X is a Banach space. We define operator A, E and their domains:

$$A = diag(-a_0, -a_1, -a_2, -\lambda_c, -\frac{d}{dx} - \mu_3(x), -\frac{d}{dx} - \mu_1(x), -\frac{d}{dx} - \mu_5(x) \quad ,$$

$$E = \begin{bmatrix} 0 & \mu_1 & 0 & \omega_3 & \omega_4 & \omega_5 \\ r_0 & 0 & \mu_2 & 0 & 0 & 0 \\ 0 & r_1 & 0 & 0 & 0 & 0 \\ \lambda_c & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \omega_k \cdot = \int_0^\infty \mu_k(x)\cdot dx, k = 3,4,5 \quad .$$

$$D(A) = \left\{ P \in X \left| \frac{dp_k(x)}{dx} \in L^1(R^+), p_3(0) = r_2 p_2(t), p_4(0) = \sum_{i=0}^{2} \lambda_{c_i} p_i(t)(i = 0,1,2; k = 3,4,5), \right. \right.$$
$$\left. p_5(0) = \lambda_c p_c(t) + \sum_{i=0}^{2} \lambda_{h_i} p_i(t) \right\}$$

Then the system (1.1) can be expressed as an Abstract Cauchy Problem in Banach space X :

$$\begin{cases} \frac{d\vec{p}(t)}{dt} = (A+E)\vec{p}(t), t > 0 \\ \vec{p}(0) = (1,0,\ldots,0)^T, t = 0 \end{cases}$$

Where: $\vec{p}(t) = (p_0(t), p_1(t), p_2(t), p_3(x,t), p_4(x,t), p_5(x,t))$

2 STEADY STATE SOLUTION OF THE SYSTEM

We first consider the existing problems of the nonzero solution about $[\gamma I - (A+E)]p = 0$,

$$\begin{cases} (\gamma + a_0)p_0 - \mu_1 p_1 - \sum_{k=3}^{5}\int_0^\infty P_k(x,t)\mu_k(x)dx = 0 \\ -r_0 p_0 + (\gamma + a_1)p_1 - \mu_2 p_3 = 0 \\ -r_1 p_1 + (\gamma + a_2)p_2 = 0 \\ -\lambda_c p_1 + (\gamma + a_3)p_3 = 0 \\ \left[\gamma + \frac{d}{dx} + \mu_k(x)\right] P_k(x,t) = 0, k = 3,4,5 \end{cases}$$

Solution of the equation group is $P_k(x,t) = P_k(0,t)e^{-\int_0^x (\gamma + \mu_k(\xi))d\xi}, k = 3,4,5$.

Take $P_k(x,t)$ substituted into the first function of the equation group, we get

$$(\gamma+a_0)p_0-\mu_1 p_1-\sum_{k=3}^{5}P_k(0,t)\int_0^\infty \mu_k(x)e^{-\int_0^x(\gamma+\mu_k(\xi))d\xi}dx=0$$

denote $A_k=\int_0^\infty \mu_k(x)e^{-\int_0^x(\gamma+\mu_k(\xi))d\xi}dx$, and combine boundary value problem, we get

$$\begin{cases}(\gamma+a_0)p_0-\mu_1 p_1-\sum_{k=3}^{5}\int_0^\infty P_k(x,t)\mu_k(x)dx=0\\ -r_0 p_0+(\gamma+a_1)p_1-\mu_2 p_3=0\\ -r_1 p_1+(\gamma+a_2)p_2=0\\ -\lambda_c p_1+(\gamma+a_3)p_3=0\end{cases}\qquad (2.2)$$

denote the coefficient determinant,

$$D(\gamma)=\begin{vmatrix}\gamma+a_0 & -\mu_0 & 0 & A_3 & A_4 & A_5\\ -r_0 & \gamma+a_1 & 0 & -\mu_2 & 0 & 0\\ 0 & -r_1 & \gamma+a_2 & 0 & 0 & 0\\ -\lambda_c & 0 & 0 & \gamma+a_3 & 0 & 0\\ 0 & 0 & 0 & 0 & -1 & 0\\ \lambda_{c_0} & \lambda_{c_1} & \lambda_{c_2} & \lambda_{c_3} & 0 & -1\end{vmatrix}$$

when $\gamma\in C$ is the eigenvalues of $A+E$,then $D(\gamma)=0$;conversely when $D(\gamma)=0$,the equation group(2.2) have the nonzero solution $(p_0,p_1,p_2,p_3,p_4(x),p_5(x))$,

denote $B_k=e^{-\int_0^x(\gamma+\mu_k(\xi))d\xi}$, then $(p_0,p_1,p_2,p_3,p_4(0)B_4,p_5(0)B_5)\in D(A+E)$ is one solution of the equation group(2.1), specifically when $\gamma=0$, $A_k=1$ then $D(\gamma)=0$, that $\gamma=0$ is an eigenvalue of $A+E$, and the corresponding eigenvector $P=(p_0,p_1,p_2,p_3,p_4(x),p_5(x))$, where $b_1=\frac{(\gamma+a_0)}{\mu_1}, b_2=\frac{(\gamma+a_0)(\gamma+a_1)}{(\gamma+a_2)\mu_1}, b_3=\frac{(\gamma+a_0)(\gamma+a_1)(\gamma+a_2)}{(\gamma+a_2)\mu_1\mu_2}, b_4=0$,

Let $Q=(1,\cdots,1)$, then $<P,Q>=p_0+p_1+p_2+p_3+\int_0^\infty p_4(x)dx+\int_0^\infty p_5(x)dx>0$ for all $P\in D(A+E)$, we have $<(A+E)P,Q>=0$, $(A+E)Q=0$, so we obtain that 0 is a simple eigenvalue of the operator $A+E$, standardized the eigenvector, we get

$$\hat{p}_0=\frac{1}{N}p_0,\ \hat{p}_1=\frac{b_1}{N}p_0,\ \hat{p}_2=\frac{b_2}{N}p_0,\ \hat{p}_3=\frac{b_3}{N}p_0,\ \hat{p}_4(x)=\frac{b_4B_4}{N}p_0,$$

$$P_5(x)=\frac{1}{N}[\lambda_{c_0}+\lambda_{c_1}b_1+\lambda_{c_2}b_2+\lambda_{c_3}b_3]B_5P_0,$$

Where, $N=(1+b_1+b_2+b_3)+b_4B_4+(\lambda_{c_0}+\lambda_{c_1}b_1+\lambda_{c_2}b_2+\lambda_{c_3}b_3)B_5$. Then we obtain the steady state solution of the system $\hat{p}=(\hat{p}_0,\hat{p}_1,\hat{p}_2,\hat{p}_3,\hat{p}_4(x),\hat{p}_5(x))$, Steady state availability:

$$A=\frac{\hat{p}_0+\hat{p}_1+\hat{p}_2}{\hat{p}_0+\hat{p}_1+\hat{p}_2+\int_0^\infty \hat{p}_3(x)dx+\int_0^\infty \hat{p}_4(x)dx+\int_0^\infty \hat{p}_5(x)dx}=\frac{1+f}{N}$$

where:

$$f=\frac{a_0}{\mu_1}+\frac{-r_0\mu_1+a_0a_1}{\mu_1\mu_2},\quad N=1+\frac{a_0}{\mu_1}+\frac{-r_0\mu_1+a_0a_1}{\mu_1\mu_2}+\frac{-r_0\mu_1+a_0a_1}{\mu_1\mu_2}r_0\int_0^\infty e^{-\int_0^x\mu_3(\xi)d\xi}dx+$$

$$(\lambda_{c_0}+\frac{a_0}{\mu_1}\lambda_{c_1}+\frac{-r_0\mu_1+a_0a_1}{\mu_1\mu_2}\lambda_{c_2})\int_0^\infty e^{-\int_0^x\mu_4(\xi)d\xi}dx+(\lambda_{h_0}+\frac{a_0}{\mu_1}\lambda_{h_1}+\frac{-r_0\mu_1+a_0a_1}{\mu_1\mu_2}\lambda_{h_2})\int_0^\infty e^{-\int_0^x\mu_5(\xi)d\xi}dx,$$

$$A_1=\frac{\hat{p}_0+\hat{p}_1+\hat{p}_2+\hat{p}_c}{\hat{p}_0+\hat{p}_1+\hat{p}_2+\hat{p}_c+\int_0^\infty\hat{p}_3(x)dx+\int_0^\infty\hat{p}_4(x)dx+\int_0^\infty\hat{p}_5(x)dx}=\frac{1+f_1}{N_1}$$

Where:

$$f_1=\frac{a_0}{\mu_1}+\frac{-r_0\mu_1+a_0a_1}{\mu_1\mu_2}+\frac{\lambda_{c_0}}{\lambda_c}\quad,N_1=1+\frac{a_0}{\mu_1}+\frac{-r_0\mu_1+a_0a_1}{\mu_1\mu_2}+\frac{\lambda_{c_0}}{\lambda_c}+\frac{-r_0\mu_1+a_0a_1}{\mu_1\mu_2}r_0\int_0^\infty e^{-\int_0^x\mu_3(\xi)d\xi}dx+$$

$$(\lambda_{c_0}+\frac{a_0}{\mu_1}\lambda_{c_1}+\frac{-r_0\mu_1+a_0a_1}{\mu_1\mu_2}\lambda_{c_2})\int_0^\infty e^{-\int_0^x\mu_4(\xi)d\xi}dx+(\lambda_{h_0}+\frac{a_0}{\mu_1}\lambda_{h_1}+\frac{-r_0\mu_1+a_0a_1}{\mu_1\mu_2}\lambda_{h_2})\int_0^\infty e^{-\int_0^x\mu_5(\xi)d\xi}dx.$$

Suppose $r_0=r_1=r_2=\lambda_{c_0}=\lambda_{c_1}=\lambda_{c_2}=\lambda_{h_0}=\lambda_{h_1}=\lambda_{h_2}=\lambda\,,\lambda_c=\alpha\lambda$,

$\mu_1=\mu_2=\mu_3(x)=\mu_4(x)=\mu_5(x)=\mu$, the results of the simulation of the above conclusions by using Mathematica software are shown in table 1 to table 3.

It can be seen from the results that when other parameters do not change, with the increase of α , the relative error of steady-state availability tends to zero.

TABLE 1 RELATIVE ERROR VARIATION AT μ =1

	λ	μ	α	A_1	A	Error
1	0.003	1	0.1	0.999446	0.994018	0.005428
2	0.003	1	1	0.996978	0.994018	0.002960
3	0.003	1	10	0.994552	0.994018	0.000534
4	0.003	1	100	0.994076	0.994018	0.000058
5	0.003	1	1000	0.994024	0.994018	0.000006
6	0.003	1	1000	0.994019	0.994018	0.000000

TABLE 2 RELATIVE ERROR VARIATION AT μ =0.1

	λ	μ	α	A_1	A	Error
1	0.003	0.1	0.1	0.993630	0.941829	0.051801
2	0.003	0.1	1	0.967919	0.941829	0.026090
3	0.003	0.1	10	0.946204	0.941829	0.004375
4	0.003	0.1	100	0.942298	0.941829	0.000469
5	0.003	0.1	1000	0.941876	0.941829	0.000047
6	0.003	0.1	1000	0.941833	0.941829	0.000000

TABLE 3 RELATIVE ERROR VARIATION AT μ =0.01

	λ	μ	α	A_1	A	Error
1	0.003	0.01	0.1	0.846746	0.578773	0.267973
2	0.003	0.01	1	0.641465	0.578773	0.062692
3	0.003	0.01	10	0.586011	0.578773	0.007238
4	0.003	0.01	100	0.579508	0.578773	0.000735
5	0.003	0.01	1000	0.578846	0.578773	0.000073
6	0.003	0.01	1000	0.578774	0.578773	0.000000

Through the simulation results from above, we can see that the relative error tends to steady state availability with $\alpha \to \infty$ if other conditions do not change. In practice, according to the practical significance of risk coefficient tends to infinite, the conclusions reflect the relationship between the early warning system and the non-warning system.

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