

Research of Physical Parameter Identification and Damage Localization based on the Gibbs Sampling

Ziyan Wu¹, Zongming Cai^{1†}, Shukui Liu²

1.School of Mechanics, Civil Engineering and Architecture, Northwestern Polytechnical University, Xi'an 710129, China

2.School of Mechanics and Civil Engineering, China University of Mining and Technology, Xuzhou 221116, China

†Email: caizongming1992@163.com

Abstract

A new method for structural physical parameter identification is proposed for linear structure. Firstly, a linear structural identification model was obtained based on a series of transformation of the dynamic characteristic equation. Then the posterior distribution of the model is obtained by the Bayesian updating theory. Using the structural modal parameters and considering their randomness, the structural stiffness parameter is obtained from the conditional posterior distribution of the linear structural identification model. The Gibbs sampling based on the Markov Chain Monte Carlo (MCMC) method is employed during the process. In order to illustrate the proposed method, a 3-DOF linear shear building is used as an example to detect and quantify its damage based on model data measured before and after a severe loading event. The research shows that damage level and locations can be identified with little error by using proposed method.

Keywords: *Physical Parameters Identification; Damage Localization; MCMC Method; Gibbs Sampling*

1 INTRODUCTION

In recent years, the problem of structural physical parameters identification for a system has attracted the attention of many researchers, Gong and Li^[1] adopted Bayesian statistics theory and the Markov Chain Monte Carlo (MCMC) sample method to estimate the structural physical parameters, Yuen and Mu^[2] proposed a novel Bayesian real-time system identification algorithm using response measurement for dynamical systems. But under the influence of uncertain factors, such as non-uniformity in material properties, variability in complex constitutive behavior and randomness in the excitations, uncertainty becomes the essential characteristic of measurement data and structural analysis model, which makes structural physical parameter identification an indeterminate problem^[3]. So new approaches capable of identifying structural physical parameters from a probabilistic point of view are needed. Considering the uncertainties, Ito and Kogiso^[4] adopted the Reliability-based design optimization(RBDO) method to estimate the parameters.

In the field of civil engineering, Beck and Katafygiotis^[5] proposed a Bayesian structural model updating approach at first, but the amount of modal data must be sufficiently large. Beck and Au^[6] proposed a stochastic simulation approach to overcome this problem, based on a Markov chain Monte Carlo algorithm (Metropolis-Hastings). However, a major limitation is that it is only efficient for lower-dimensional problems. Ching and Cheng^[7] proposed the transitional Markov chain Monte Carlo (TMCMC) algorithm to overcome this limitation, while the Markov chains must “burn-in” a sufficiently long time. The Gibbs sampling can solve high-dimensional model updating problems effectively^[8], Li and Xie^[3] adopted this approach to identify structural physical parameters using certain modal data, but due to the environmental impact, test error and the simplified analytical process, the uncertainties exist inevitably between the measured modal parameters and the true values^[9].

In this paper, a new structural physical parameters identification approach is presented for linear structural models. At first, a linear structural identification model was obtained based on a series of transformation of the dynamic

characteristic equation. Then the posterior distribution of the model is obtained by the Bayesian updating theory. Using the structural modal parameters and considering their randomness, the structural stiffness parameter is obtained from the conditional posterior distribution of the linear structural identification model. The Gibbs sampling based on the Markov Chain Monte Carlo method is employed during the process.

2 THE LINEAR REGRESSION MODEL

The linear regression model is generally represented as follows ^[10] (Scott, 2007):

$$Y = X\theta + e \quad (1)$$

$$e \sim N(0, \sigma_e^2 I) \quad (2)$$

where y_i is equal to a linear combination of a set of predictors, $X_i^T \theta$ plus error e_i , and that the error term is normally distributed with a mean of 0 and some variance σ_e^2 , and I is an n -dimensional identity matrix. The diagonal elements of this matrix are all equal, and the off-diagonal elements of this matrix are 0s.

A Bayesian specification typically begins with a normality assumption on $y|x$ (often with the conditioning suppressed): $y_i \sim N(X_i^T \theta, \sigma_e^2)$. And an improper uniform prior over the real line is often specified for the regression parameters θ , namely:

$$P(\theta_i) = 1 \quad (\theta_i \in (0, +\infty); i = 1, 2, \dots, m) \quad (3)$$

The prior probability distribution function (PDF) for σ_e^2 is taken to be the product of independent inverse gamma PDFs,

$$P(\sigma_e^2) \sim IG(\alpha, \beta) \propto (1/\sigma_e^2)^{\alpha+1} e^{-(\beta/\sigma_e^2)} \quad (4)$$

When $\alpha = \beta = 0$, the inverse gamma prior becomes the usual Jeffreys' non-informative prior, i.e.,

$$P(\sigma_e^2) \sim 1/\sigma_e^2 \quad (5)$$

This yields a posterior distribution that appears as:

$$P(\theta, \sigma_e^2 | X, Y) \propto (\sigma_e^2)^{-(n/2+1)} \exp \left\{ -\frac{1}{2\sigma_e^2} (Y - X\theta)^T (Y - X\theta) \right\} \quad (6)$$

A Gibbs sampler for the linear regression model can be developed when the full conditional posterior distribution of θ and σ_e^2 is known. The full conditional posterior distribution for σ_e^2 is straightforward to derive from Eq. (6):

$$P(\sigma_e^2 | \theta, X, Y) \propto (\sigma_e^2)^{-(n/2+1)} \exp \left\{ -\frac{1}{2\sigma_e^2} (Y - X\theta)^T (Y - X\theta) \right\} \quad (7)$$

This conditional posterior is easily seen to be an inverse gamma distribution with parameters $\alpha = n/2$ and $\beta = (Y - X\theta)^T (Y - X\theta)/2$. While the conditional posterior distribution for θ is:

$$P(\theta | \sigma_e^2, X, Y) \propto (\sigma_e^2)^{-(n/2+1)} \exp \left\{ -\frac{1}{2\sigma_e^2 (X^T X)^{-1}} [\theta^T \theta - 2\theta^T (X^T X)^{-1} (X^T Y)] \right\} \quad (8)$$

It can be recognized that the conditional posterior distribution for θ is normal with a mean equal to $(X^T X)^{-1} (X^T Y)$ and a variance of $\sigma_e^2 (X^T X)^{-1}$.

3 LINEAR STRUCTURAL IDENTIFICATION MODEL

In terms of structural health monitoring, linear structural models are often used for model updating, since much vibration data of structures under investigation are obtained using low-amplitude excitation, such as ambient vibration and hammer. In this case, many structures (even damaged nonlinear structures) behave approximately linearly, so the linearity assumption of the approach is justified ^[8].

The i th natural frequency ω_i and mode shape vector ϕ_i of a n DOF system satisfy the following characteristic

equation:

$$[K - \omega_i^2 M]\phi_i = \{0\} \quad (9)$$

The expanded form of the equation can be expressed as follows:

$$\omega_i^2 \begin{bmatrix} m_1 & & & \\ & m_2 & & \\ & & \ddots & \\ & & & m_n \end{bmatrix} \begin{Bmatrix} \phi_{i1} \\ \phi_{i2} \\ \vdots \\ \phi_{in} \end{Bmatrix} = \begin{bmatrix} k_1\theta_1 + k_2\theta_2 & & & -k_2\theta_2 \\ & -k_2\theta_2 & & \\ & & k_2\theta_2 + k_3\theta_3 & \ddots \\ & & & \ddots \\ & & & & -k_n\theta_n \\ & & & & k_n\theta_n & \\ & & & & k_n\theta_n & \end{bmatrix} \begin{Bmatrix} \phi_{i1} \\ \phi_{i2} \\ \vdots \\ \phi_{in} \end{Bmatrix} \quad (10)$$

Where $\theta_1, \theta_2, \dots, \theta_n$ normalized no-dimensional parameters (calling the structural stiffness parameters in the following). θ_i indicates the contribution ratio of one structural member to the whole structure, and values range from 0 to 1. The i th structural member can be judged to be damaged when $\theta_i < 1$, and the damage degree can be seen through the value of θ_i . Note that the mass of the structural members are fixed values since it is less sensitive to the damage. If there is a need to identify the mass parameters, a similar transform to equation (9) will do.

Transform Eq. (10) and plus the error term, then the linear structural identification model can be expressed as follow:

$$\omega_i^2 \begin{bmatrix} m_1 & & & \\ & m_2 & & \\ & & \ddots & \\ & & & m_n \end{bmatrix} \begin{Bmatrix} \phi_{i1} \\ \phi_{i2} \\ \vdots \\ \phi_{in} \end{Bmatrix} = \begin{bmatrix} k_1\phi_{i1} & k_2(\phi_{i1} - \phi_{i2}) & & & \\ & -k_2(\phi_{i1} - \phi_{i2}) + k_3\phi_{i2} & & & \\ & & \ddots & & \\ & & & -k_{n-1}(\phi_{i(n-2)} - \phi_{i(n-1)}) + k_n\phi_{i(n-1)} & -k_n\phi_{in} \\ & & & & -k_n(\phi_{i(n-1)} - \phi_{in}) \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{Bmatrix} + e \quad (11)$$

4 GIBBS SAMPLER ALGORITHM

- Draw the initial sample $\{\hat{\theta}^{(0)}, \hat{\sigma}_e^{2(0)}\}$ from the prior PDFs and let $k = 1$;
- According to Eq. (7) and (11), sample the error term $\hat{\sigma}_e^{2(k)} \sim P(\sigma_e^2 | \hat{\theta}^{(k-1)}, X, Y)$;
- According to Eq. (8) and (11), sample the structural stiffness parameters $\theta^{(k)} \sim P(\theta | \hat{\sigma}_e^{2(k)}, X, Y)$;
- Let $k = k + 1$, go back to step two and cycle to obtain N samples $\{\hat{\theta}^{(k)}, \hat{\sigma}_e^{2(k)} : k = 1, 2, \dots, N\}$.

When k gets large enough, the samples $\{\hat{\theta}^{(k)}, \hat{\sigma}_e^{2(k)}\}$ will follow the PDF $P(\theta, \sigma_e^2 | X, Y)$.

5 DAMAGE PROBABILITY EVALUATION

Since a one to one correspondence exists between the elements of structural stiffness parameters vector θ and the components of the structure, so in this paper, the damage will be defined as a reduction in the value of any element of the structural stiffness parameters vector θ below some specified threshold value. And estimated the damage probability will be defined to be that any element of the structural stiffness parameters vector θ has decreased more than a fraction d .

Note that the samples $\{\hat{\theta}^{(k)}, \hat{\sigma}_e^{2(k)}\}$ will ultimately distribute as $P(\theta, \sigma_e^2 | X, Y)$, implying that the extracted stiffness samples $\{\hat{\theta}^{(k)} : k = 1, 2, \dots, N\}$ will ultimately distribute as $P(\theta | X, Y)$.

After sampling from both the undamaged and damaged structures, the damage probability may be approximated using M comparisons between the values of samples generated from the two sets of data, as follows (Muto, 2006):

$$P(\theta_i^{pd} < (1-d)\theta_i^{ud} | X^{ud}, Y^{ud}, X^{pd}, Y^{pd}) \approx \frac{1}{M} \sum_{n=1}^M I[\hat{\theta}_{in}^{pd} < (1-d)\hat{\theta}_{in}^{ud}] \quad (12)$$

where $I[\cdot]$ is the indicator function, which is unity when the condition is satisfied and zero otherwise, the samples $\hat{\theta}_{in}^{ud}$ and $\hat{\theta}_{in}^{pd}$ are chosen randomly from the available samples conditioned on the undamaged and possibly damaged data, respectively. And M is the sample size after the “burn-in” period. In order to make the results accurate enough, M needs to be a large integer. In this paper the “burn-in” period is taken to be 1000, and M is chosen to be 4000.

6 NUMERICAL EXAMPLE

To examine the performance of the Gibbs sampler algorithm, studies are performed using simulated data from a 3-DOF linear shear structure, in which the height, mass and stiffness are chosen to be $h_1=5\text{m}$, $h_2=h_3=4\text{m}$; $m_1=2000\text{kg}$, $m_2=1500\text{kg}$, $m_3=1000\text{kg}$; $k_1=1800\text{kN/m}$, $k_2=1200\text{kN/m}$, $k_3=600\text{kN/m}$. Damage is simulated by reducing the column stiffness, as shown in Fig.1. There are three damage patterns in this paper: (1) DP1: loss of 30% column stiffness in the first floor; (2) DP2: loss of 30% column stiffness in the second floor and loss of 50% column stiffness in the top floor; (3) DP3: the losses of column stiffness in the first, second and top floor are 50%, 40%, 30%, respectively. In order to take the randomness of the modal parameters into consideration, in this paper, the first-order natural frequency used during the Gibbs sampling process is assumed to follow a normal distribution with mean taken to be the theory natural frequency value and proper variance, as shown in Table 1.

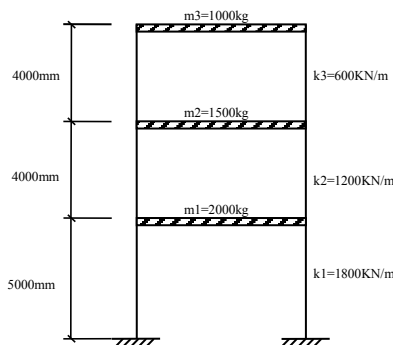


FIG.1 THE 3-DOF LINEAR SHEAR STRUCTURE

TABLE 1. STATISTICAL PROPERTIES OF THE FIRST-ORDER NATURAL FREQUENCY

Damage Patterns	Undamaged(UD)	DP1	DP2	DP3
Mean of ω_1 (rad/s)	14.51	13.30	12.34	10.91
Variance of ω_1	1.02	1.06	1.17	0.93

For simplicity, the initial samples of the structural stiffness parameters vector θ of the four patterns are all taken to be $\theta = [0.8 \quad 0.8 \quad 0.8]^T$. Following the Gibbs sampler algorithm, the Markov chain samples of the structural stiffness parameters are obtained. In Fig.2, all the samples of θ_1 for the undamaged and damaged case are plotted (to save space, the samples of θ_2 and are omitted). Fig.2 shows clearly that when any structural component was damaged, the corresponding Markov chain samples curve will change obviously, making it easy to locate the damage position and identify the degree of damage.

In Fig.3 the Markov chain samples in different parameter space for all damaged patterns are plotted. The results show approximate linearity for the relations between different parameters, which imply that the results are reasonable. It is also very easy to locate the damage position and identify the degree of damage from Fig.3. For example, in Fig.3.1, compare with the curve marked with “o” (UD), the curve marked with “+” (DP1) shifts

distinctly to the left (θ_1 direction) while there is almost no shift in θ_2 direction. This implies that there is a loss of stiffness in the column k_1 and there is no damage in column k_2 , just be consistent with the damage condition we assumed before. It should be noticed that the curve marked with “o” (UD) and the curve marked with “+”(DP1) overlap closely in Fig.3.3, this is because that Fig.3.3 is plotted in the (θ_2, θ_3) space while in DP1 there is no damage assumed in k_2 and k_3 .

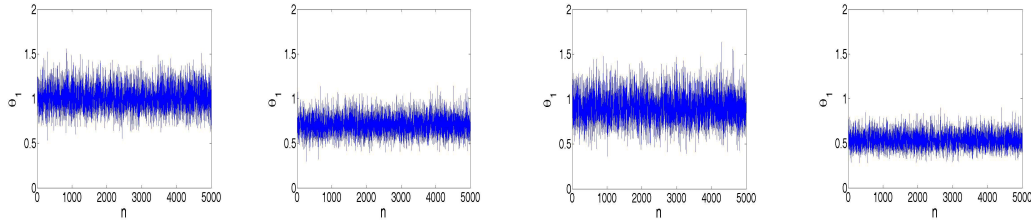


FIG.2 SAMPLES FOR THE STRUCTURAL STIFFNESS PARAMETERS FROM DIFFERENT DAMAGED PATTERNS

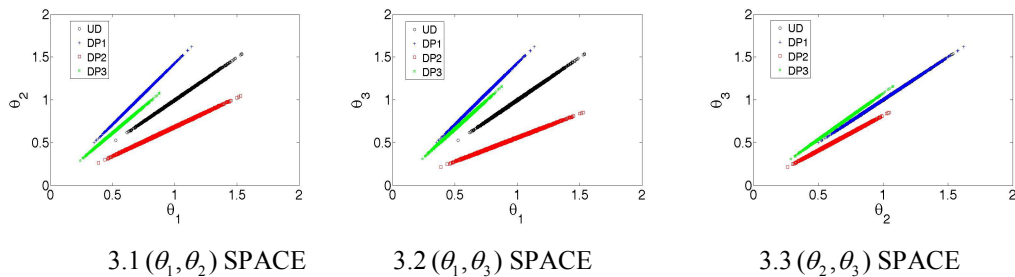


FIG.3 STRUCTURAL STIFFNESS PARAMETERS SAMPLES FOR ALL DAMAGED PATTERNS PLOTTED IN DIFFERENT SPACES

Fig.4 shows the probability that the loss of stiffness of different columns exceeds the threshold d for different damaged patterns, estimated using Eq. (12). Through this approach, it cannot just locate the damage position and identify the degree of damage clearly, but update the associated uncertainties, just the advantages of Bayesian techniques.

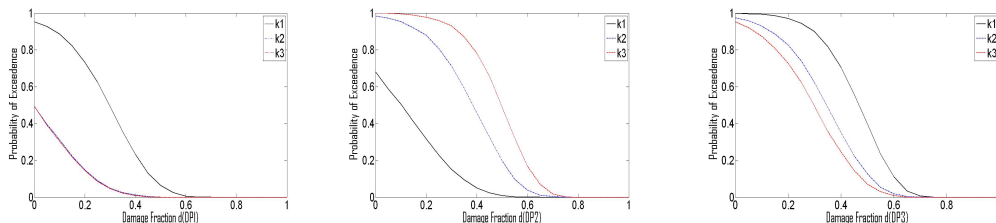


FIG.4 ESTIMATED DAMAGE PROBABILITY CURVES FOR ALL DAMAGED PATTERNS

TABLE 2. POSTERIOR STATISTICAL PROPERTIES OF THE STIFFNESS PARAMETERS

Damaged Patterns	θ_1			θ_2			θ_3		
	Mean	Var	Error	Mean	Var	Error	Mean	Var	Error
UD	1.0065	0.1407	0.65%	1.0033	0.1402	0.33%	1.0031	0.1402	0.31%
DP1	0.7058	0.1131	0.83%	1.0081	0.1615	0.81%	1.0082	0.1616	0.82%
DP2	0.9047	0.1675	9.53%	0.6184	0.1144	11.66%	0.5050	0.0935	1.00%
DP3	0.5347	0.0922	6.94%	0.6552	0.1131	9.20%	0.7043	0.1216	0.61%

Table 2 shows the posterior statistical properties of the stiffness parameters. Since only the first-order natural frequency is used and the uncertainties in the natural frequency data, when the degree of damage increases, the error of identified results also increases slightly, but within an acceptable range.

7 CONCLUSION

A new structural physical parameters identification approach is presented for linear structural models. The results from the simulated 3-DOF linear shear building show that the proposed approach can not only identify the damage

degree and locations in different ways with little error, but also inherits the advantages of Bayesian techniques: it updates both the optimal estimate of the structural parameters and the associated uncertainties.

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AUTHORS



¹**Ziyang Wu**(1962-),female,Ph.D., Professor, Doctoral supervisor, mainly engaged the research of structural health monitoring and reliability assessment, graduated from Northwestern Polytechnical University in 2006, acquired PhD.

Email: zywu@nwpu.edu.cn.

²**Zongming Cai** (1992-), male, Bachelor, Master candidate, mainly focused on performance-based design and fragility

analysis, graduated from Northwestern Polytechnical University in civil engineering, acquired Bachelor Degree.

Email:caizongming1992@163.com

³**Shukui Liu** (1986-),male,Ph.D.,assistant professor of China University of Mining and Technology, mainly focused on non-destructive testing and damage identification, graduated from Northwestern Polytechnical University in 2015, acquired PhD.

Email:skliu1986@gmail.com