

# Stability Analysis of Bolt-reinforced Slopes Using of 3D DDA

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## **Abstract**

This paper provides the stability analysis of bolt-reinforced slopes using of 3D DDA (Discontinuous Deformation Analysis). Up to now, the 2D DDA has been developed maturely in theory and practice. However, the usage of 3D DDA is rare according to literature review especially in practical 3D slopes. The function of bolt reinforcement has been fulfilled in the 2D DDA, while it has not been realized and applied in practical engineering in the 3D DDA. For the practical engineering problem, obviously, the 3D analysis is more accurate. Therefore, to fulfil the function of bolt reinforcement and application in practical engineering is still a problem for the 3D DDA. In this research, firstly, some basic theory about the 3D DDA method and the CRLD (Constant Resistance and Large Deformation) bolt model will be illustrated. Secondly, how to add the function of bolt reinforcement to the 3D DDA is illustrated, and then a simple test mode is used to test its accuracy. Lastly, a slope model is used as an example to verify its usage in engineering. In this paper, the process of adding bolt function in the 3D DDA is illustrated and the verification in slope is made, which benefits the future development of 3D DDA in landslide mitigation.

**Keywords:** 3D DDA; Bolt Mechanism; CRLD

## **1 INTRODUCTION**

Discontinuous Deformation Analysis (DDA) is one of the most popular discrete element methods, which was originally created by Shi (1988). It can analyse the static and dynamic performances of jointed or blocky media, without assuming failure modes. The DDA allows the simulation of not only translation, rotation and deformation of each individual block having been widely used in the rock mass engineering. 2D DDA has been extended to 3D domain by many researchers in recent years. Modelling the rock bolt mechanism in rock masses is a difficult work. The 2D DDA has been used to simulate the bolt mechanism by some researches. Yeung (1993) presented the qualitative validation of the mining roof reinforced by rock bolts. Shi (2009) analysed the rock block topping with bolt reinforced. The 2D DDA uses the simplified linear elastic bolt model and shows the reasonable result, which can help us to make basic assessment of the block mass reinforced by bolts. During the simulation, the bolt can be treated as an elastic Newton element, which is a basic simplified model and used in the 2D DDA simulation. Recently, He (2014) proposed a new kind of bolt (He bolt), which serves constant resistance and large deformation (CRLD). This bolt has been used in engineering and shown excellent performance. However, how to simulate the bolt using 3D-DDA is still a problem. In this study, the bolt mechanism will be illustrated and how to couple it with 3D DDA code is specified. A verify example is shown. Meanwhile, a slope example will be presented.

## **2 COUPLE BOLT ALGORITHM AND 3D-DDA CODE**

### **2.1 3D DDA Method**

Performing as a Lagrangian energy-based method for discrete elements, the 3D DDA block system conforms to the principle of minimum total potential energy, which is the summation of potential energy of each block. The detail formulations can be found from Shi (2001) and Liu (2004). The sources of potential energy includes the elastic stress, the initial constant stresses, the point load, the body forces, the inertia forces, contact forces between blocks, and etc. The total potential energy of a block system with n blocks is formed:

$$\Pi = \frac{1}{2} \begin{bmatrix} \mathbf{D}_1^T & \mathbf{D}_2^T & \cdots & \mathbf{D}_n^T \end{bmatrix} \begin{bmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} & \cdots & \mathbf{K}_{1n} \\ \mathbf{K}_{21} & \mathbf{K}_{22} & \cdots & \mathbf{K}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{K}_{n1} & \mathbf{K}_{n2} & \cdots & \mathbf{K}_{nn} \end{bmatrix} \begin{bmatrix} \mathbf{D}_1 \\ \mathbf{D}_2 \\ \vdots \\ \mathbf{D}_n \end{bmatrix} + \begin{bmatrix} \mathbf{D}_1^T & \mathbf{D}_2^T & \cdots & \mathbf{D}_n^T \end{bmatrix} \begin{bmatrix} \mathbf{F}_1 \\ \mathbf{F}_2 \\ \vdots \\ \mathbf{F}_n \end{bmatrix} + \text{Constant} \quad (1)$$

where  $\mathbf{K}_{ij}$  is a  $12 \times 12$  stiffness sub-matrix of block i,  $\mathbf{D}_i$  is  $12 \times 1$  displacement sub-matrix and  $\mathbf{F}_i$  is  $12 \times 1$  load sub-matrix of Block i, respectively. The symmetries  $\mathbf{K}_{ij} = \mathbf{K}_{ji}^T$  can be seen.

By minimizing the total potential energy, all terms of the differentiations of an n blocks' system assembling the simultaneous equilibrium equations in a matrix form as below:

$$\begin{bmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} & \cdots & \mathbf{K}_{1n} \\ \mathbf{K}_{21} & \mathbf{K}_{22} & \cdots & \mathbf{K}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{K}_{n1} & \mathbf{K}_{n2} & \cdots & \mathbf{K}_{nn} \end{bmatrix} \begin{bmatrix} \mathbf{D}_1 \\ \mathbf{D}_2 \\ \vdots \\ \mathbf{D}_n \end{bmatrix} = \begin{bmatrix} \mathbf{F}_1 \\ \mathbf{F}_2 \\ \vdots \\ \mathbf{F}_n \end{bmatrix} \quad (2)$$

In which sub-matrix  $\mathbf{K}_{ii}$  depends on the material properties and  $\mathbf{K}_{ij}$  depends on the contacts between Block i and j.

## 2.2 Couple Bolt Algorithm

Consider a bolt passing through point a( $x_1 \ y_1 \ z_1$ ), and b( $x_2 \ y_2 \ z_2$ ) belonging to blocks i and j, separately. The length of the bolt is  $l = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$ . Defining the direction cosines of the blot to be  $l_x = (x_1 - x_2)/L$ ,  $l_y = (y_1 - y_2)/L$  and  $l_z = (z_1 - z_2)/L$ , respectively. If the blocks move, the integral is the blot length  $dl$  which can be expressed by

$$dl = \left[ [D_i]^T [T_i]^T \begin{Bmatrix} l_x \\ l_y \\ l_z \end{Bmatrix} \right] - \left[ [D_j]^T [T_j]^T \begin{Bmatrix} l_x \\ l_y \\ l_z \end{Bmatrix} \right] \quad (3)$$

The potential energy can be expressed as

$$\Pi_b = \frac{k}{2l} dl^2 \quad (4)$$

In which k represents the elastic stiffness of the bolt.

Minimizing the potential energy by taking the derivatives, the corresponding four sets of  $12 \times 12$  sub-matrices are obtained and added to the global force matrices in the global equilibrium equation, as shown as below

$$\frac{k}{l} [E_i] [E_i]^T \rightarrow [k_{ii}] \quad (5)$$

$$\frac{k}{l} [E_j] [E_j]^T \rightarrow [k_{jj}] \quad (6)$$

$$-\frac{k}{l} [E_i] [E_j]^T \rightarrow [k_{ij}] \quad (7)$$

$$-\frac{k}{l} [E_j] [E_i]^T \rightarrow [k_{ji}] \quad (8)$$

In the above four functions, the  $E_i$  and  $E_j$  are expressed by  $E_i = [T_i]^T [l_x \ l_y \ l_z]^T$  and  $E_j = [T_j]^T [l_x \ l_y \ l_z]^T$  separately. During the simulation, the length of bolt is updated after each time step. The limit of the bolt extension length can be set beforehand. Besides the simple elastic model, the elastic-plastic bolt model can also be used.

## 2.3 CRLD Bolt

As a kind of elastic-plastic model, the constant resistance and large deformation are two main characteristics of the

CRLD bolt, of which He bolt (2014) is one typical example. This bolt consists of mass element, spring element, and a stick-slip element. For the spring element, it can be simulated easily. The stick-slip performance can be seen in Fig. 1. The  $P_{max}$  and  $P_{min}$  represent the maximum and minimum resistance force. The constant resistance zone exists between these two values. The elongation of this kind of bolt can reach 1000mm showing the pattern of large deformation.

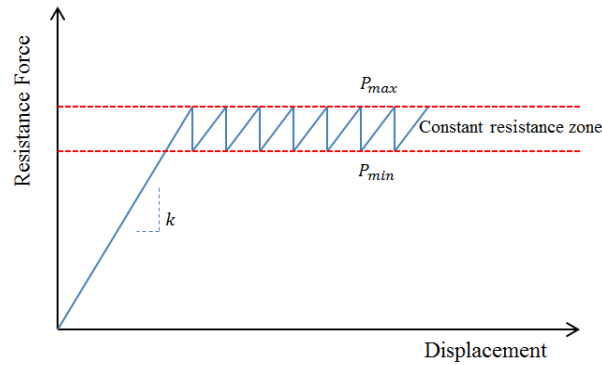


FIG. 1 FORCE-DISPLACEMENT RELATIONSHIP, AFTER HE (2014)

This paper uses a simplified model, which uses a constant resistance force. The simplified model performs like an elastoplastic model as Fig. 2 shows.

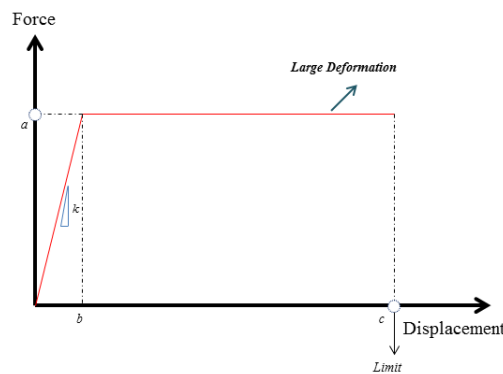


FIG. 2 SIMPLIFIED MODEL OF THE CRLD BOLT

In the Fig. 2, the point a represents the constant resistance when the elastic displacement completes, which can be seen as a tension limit. The point b is the maximum elastic displacement, c is a displacement limit, the maximum displacement of the bolt when the bolt lost its efficiency, and k is the elastic stiffness of the bolt.

### 3 SIMULATION EXAMPLES

#### 3.1 Verify Example

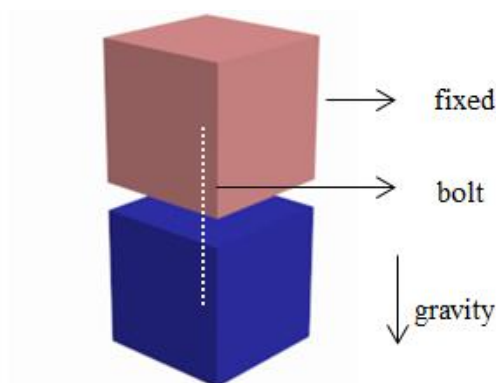


FIG. 3 THE EXAMPLE WITH TWO BLOCKS

To verify the bolt is very important before using it. A simple but direct simulation model is built containing two blocks as Fig.3 shows. Two blocks are both cubic with side length of 5 meters having a separation distance of one meter. The red block is fixed by eight fixed points at corners. The blue one owns the gravity force of 2500kN, with density of 2000 kg/m<sup>3</sup>. A bolt is added shown as write dash line, which is located at the centre of each block with a length of 6 meters.

Firstly, the simplified bolt model is completed by testifying the coincidence between the pre-set model type and the simulated result. The pre-set parameters for the He bolt model are constant resistance, bolt stiffness and displacement limit, which are 2000kN, 100Mpa and 1.5m. Using the dynamic mode, the simulated result of the relationship between the bolt displacement and bolt force can be seen from Fig. 4. We can see that the bolt mechanism behaves a well consistency with the pre-set model.

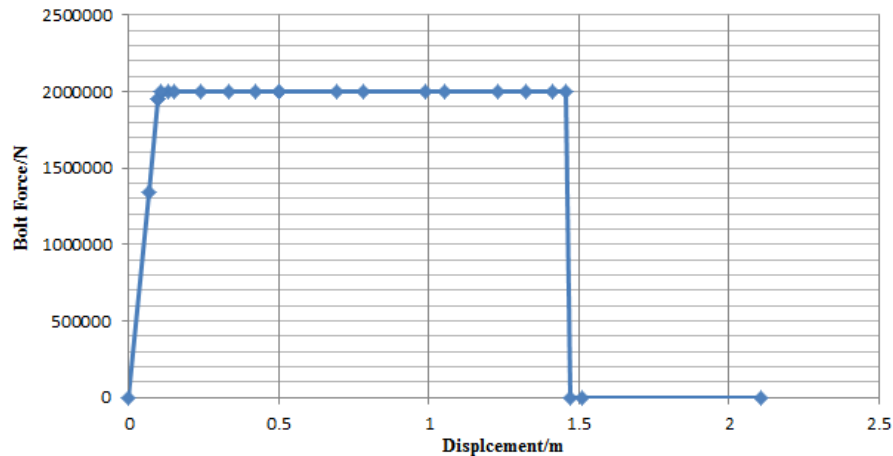


FIG. 4 THE RELATIONSHIP BETWEEN BOLT DISPLACEMENT ANF BOLT FORCE

In the above example, the below bolt will finally drop because the gravity force 2500kN is larger than the constant resistance 2000kN. If the value of bolt's constant resistance increases to be equal or larger than the gravity force, the under block will finally stop. Fig. 5 shows two kind of situation when the simulation time is one second, the left one with a constant resistance of 2200kN and right one of 2800kN.

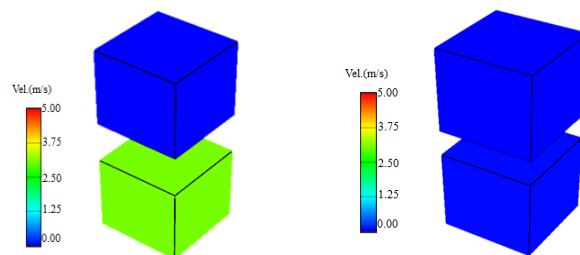


FIG. 5 COMPARE THE BOLT STATE OF DIFFERENT RESISTANT FORCE

### 3.2 Slope Example

TABLE 1 PARAMETERS OR SIMULATION

Parameters	Value
Density $\rho$ (kg/m <sup>3</sup> )	2,000
Young's modulus $E$ (GPa)	5
Poisson's ratio $\nu$	0.2
Gravitational acceleration (m/s <sup>2</sup> )	9.8
Friction angle (°)	10
Penalty spring stiffness $k$ (kN/m)	$5.0 \times 10^{10}$
Time step (s)	$10^{-3}$

A simplified slope model is built, as Fig. 6 shows. There are total 347 blocks in the simulation. The dip angle of foliation plane is  $60^\circ$ . A horizontal bolt is set inside the slope shown as a green line. Some parameters of the 3D DDA simulation are listed in Table 1.

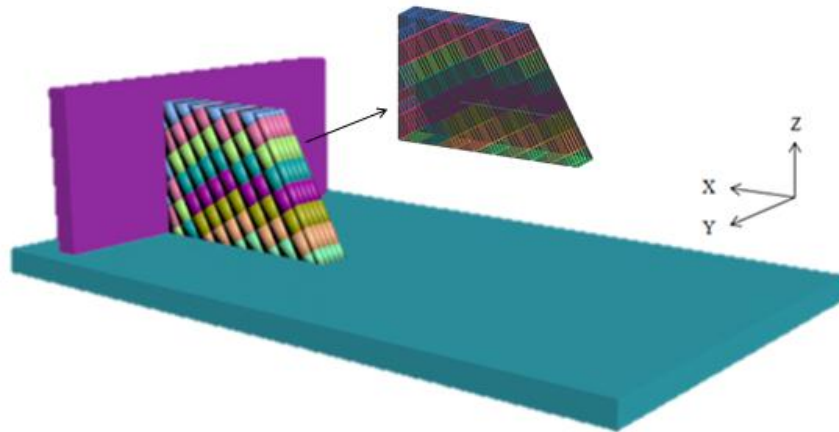


FIG. 6 GEOMETRY OF THE SIMULATION MODEL

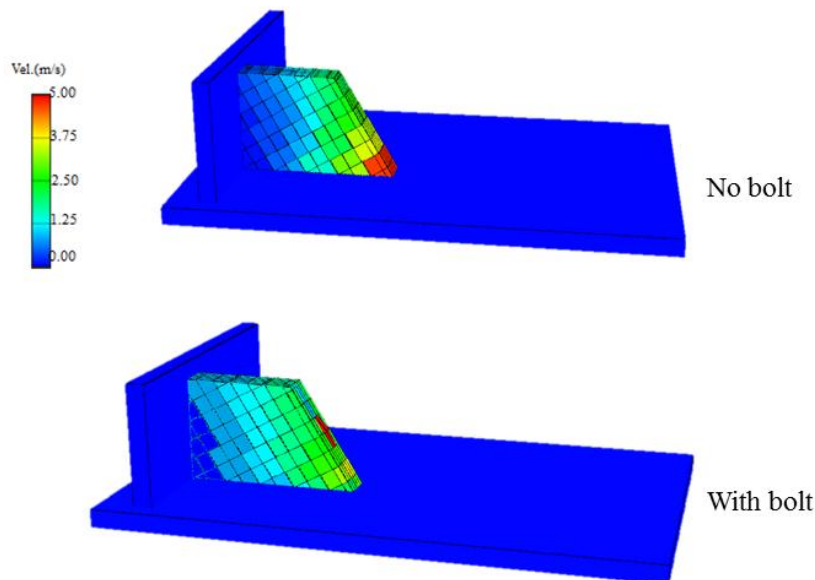


FIG. 7 SIMULATION OF THE BOLT EFFECT

Computation of three dimensional DDA uses 10000 time steps, 0.001 second per step. The dynamic ratio is 1.00. It means the next time step inherits all of the velocity from the previous time step. The slope will fail as the friction angle 10 degree is much smaller than the dip angle of the plane. The final failure situation needs a rather long time, and the simulation result just shows the state of the beginning of the failure happens. As shown in Fig. 7, the bolt has an evident influence on the deformation of the slope. The two blocks locating above the reinforced block are extruded, which does not happen when bolt is not added. The bolt has influence on the block at the toe area resulting in a different velocity pattern compared with no bolt example.

## 4 CONCLUSIONS

In this paper, the simplified bolt mechanism is achieved using the 3D DDA method. The detail information about how to couple the bolt algorithm with DDA code is provided. The CRLD bolt owning the pattern of constant resistance and large deformation is explained and used in the simulation. The verified example shows the consistency between the pre-set bolt model and simulation result. Furthermore, the slope example is used to show its potential possibility in 3D slope simulation. Further efforts should be made on at least two points: 1) using the real

bolt parameters and real 3D slope for simulation; 2) considering the distribution of load force and displacement along the bolt. It is notable that for the real simulation, these two points are necessary, but as an elementary step, some simplified assumptions are used in this paper.

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